

Introduction to Deep Learning MIT 6.S191

Alexander Amini





What is Deep Learning?

ARTIFICIAL INTELLIGENCE

Any technique that enables computers to mimic human behavior



MACHINE LEARNING

Ability to learn without explicitly being programmed



DEEP LEARNING

Learn underlying features in data using neural networks

3 1 3 4 7 2 1 7 4 4 3 5

Deep Learning Success: Vision

Image Recognition

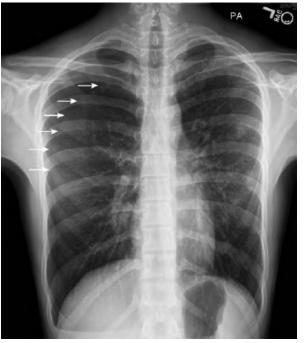


Deep Learning Success: Vision



Detect pneumothorax in real X-Ray scans







Deep Learning Success: Audio



Music Generation





Deep Learning Success

And so many more...



6.S191 Goals

Fundamentals

Practical Skills

Advancements

Community @ MIT

Knowledge, intuition, know-how, and community to do deep learning research and development

Administrative Information

- Meets Mon Jan 29 Fri Feb 2
- 10:30 am 1:30pm
- Lecture + Lab Breakdown
- Graded P/D/F; 3 Units
- I Final Assignment



Final Class Project

Option I: Proposal Presentation

- Groups of 3 or 4
- Present a novel deep learning research idea or application
- I slide, I minute (strict)
- List of example proposals on website: <u>introtodeeplearning.com</u>
- Presentations on Friday, Feb 2
- Submit groups by Wednesday
 5pm to be eligible
- Submit slide by Thursday 9pm to be eligible

- Judged by a panel of industry judges
- Top winners are awarded:





2x NVIDIA Titan TX2 MSRP: \$600



3x Google Home MSRP: \$300

Ix NVIDIA Titan Xp MSRP: \$1200

Final Class Project

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Option 2: Write a 1-page review of a deep learning paper

- Suggested papers listed on website: <u>introtodeeplearning.com</u>
- Grade is based on clarity of writing and technical communication of main ideas
- Due **Friday I 0:30am** (before lecture)

Class Support

- Piazza: https://piazza.com/class/iwmlwep2fnd5uu
- Course Website: http://introtodeeplearning.com
- Lecture slides: http://introtodeeplearning.com/#schedule
- Email us: introtodeeplearning-staff@mit.edu
- Office Hours by request



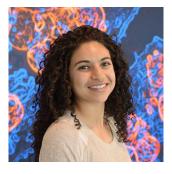
Course Staff



Alexander Amini Lead Organizer



Harini Suresh Co-Chair



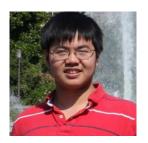
Ava Soleimany Lead Organizer



Lex Fridman Co-Chair



Shraman



Yilun



Abhimanyu



Jose Javier



Alex



Nikhil



Faraaz



Tim



Bhaktipriya

introtodeeplearning-staff@mit.edu

Thanks to Sponsors!







IBM **Research**



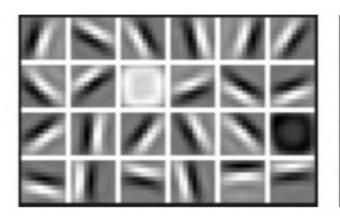
Why Deep Learning and Why Now?

Why Deep Learning?

Hand engineered features are time consuming, brittle and not scalable in practice

Can we learn the **underlying features** directly from data?

Low Level Features



Lines & Edges

Mid Level Features



Eyes & Nose & Ears

High Level Features



Facial Structure

Why Now?

1952

1958

•

1986

1995

•

Stochastic Gradient Descent

Perceptron

Learnable Weights

Backpropagation

Multi-Layer Perceptron

Deep Convolutional NN

Digit Recognition

Neural Networks date back decades, so why the resurgence?

I. Big Data

- Larger Datasets
- EasierCollection &Storage







2. Hardware

- Graphics Processing Units (GPUs)
- Massively Parallelizable



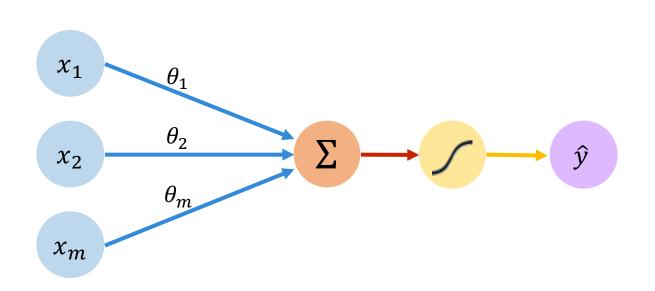
3. Software

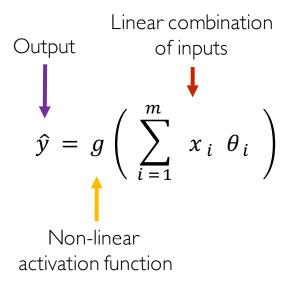
- Improved Techniques
- New Models
- Toolboxes



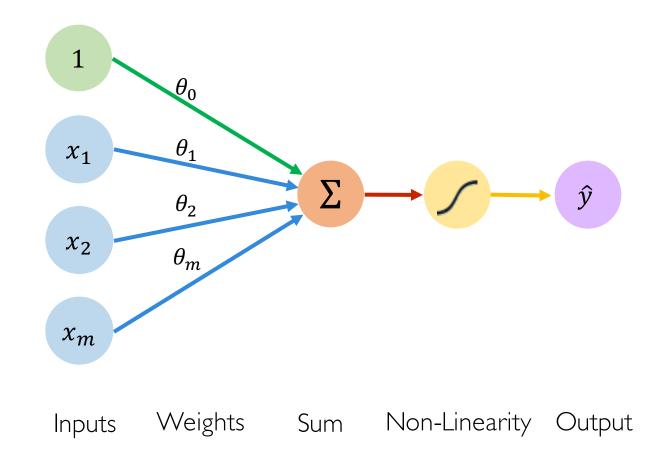


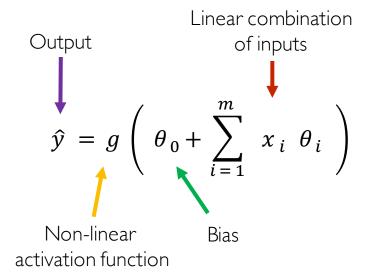
The Perceptron The structural building block of deep learning

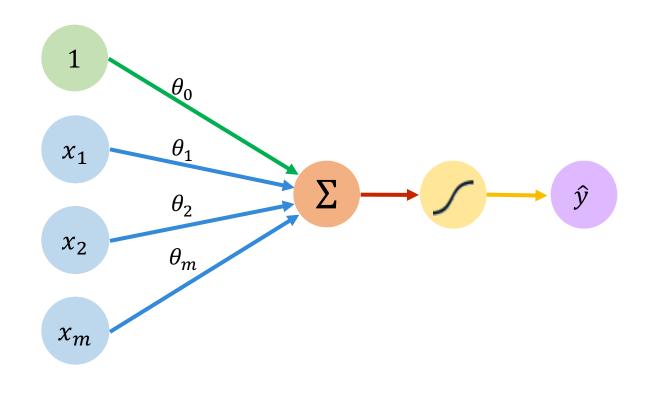




Inputs Weights Sum Non-Linearity Output







$$\hat{y} = g \left(\theta_0 + \sum_{i=1}^m x_i \theta_i \right)$$

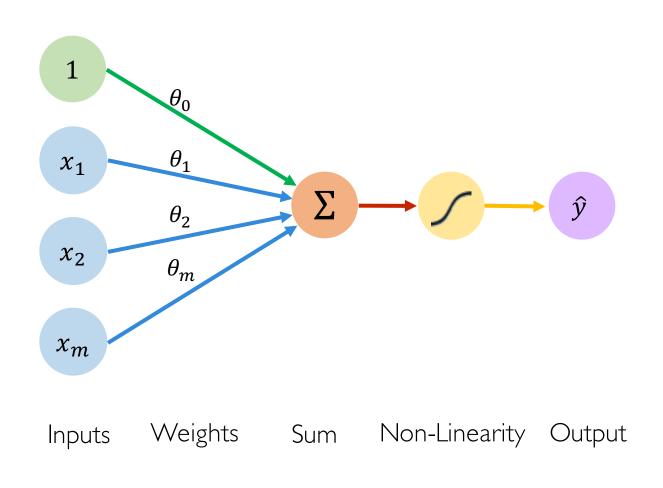
$$\hat{y} = g \left(\theta_0 + \boldsymbol{X}^T \boldsymbol{\theta} \right)$$

where:
$$\mathbf{X} = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix}$$
 and $\mathbf{\theta} = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_m \end{bmatrix}$

Weights

Sum

Non-Linearity Output

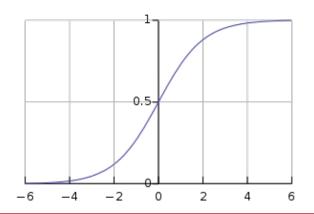


Activation Functions

$$\hat{y} = g \left(\theta_0 + X^T \theta \right)$$

• Example: sigmoid function

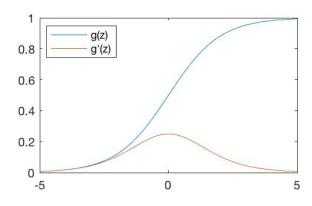
$$g(z) = \sigma(z) = \frac{1}{1 + e^{-z}}$$



 \boldsymbol{Z}

Common Activation Functions

Sigmoid Function

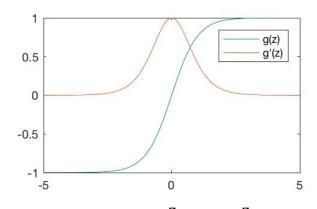


$$g(z) = \frac{1}{1 + e^{-z}}$$

$$g'(z) = g(z)(1 - g(z))$$



Hyperbolic Tangent

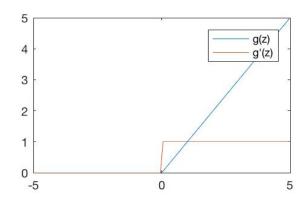


$$g(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

$$g'(z) = 1 - g(z)^2$$



Rectified Linear Unit (ReLU)



$$g(z) = \max(0, z)$$

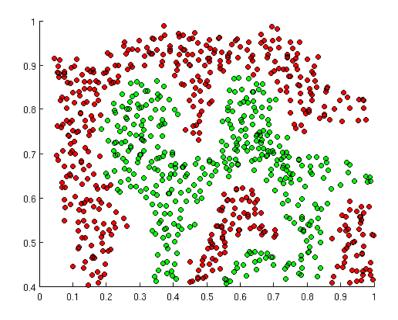
$$g'(z) = \begin{cases} 1, & z > 0 \\ 0, & \text{otherwise} \end{cases}$$



NOTE: All activation functions are non-linear

Importance of Activation Functions

The purpose of activation functions is to **introduce non-linearities** into the network

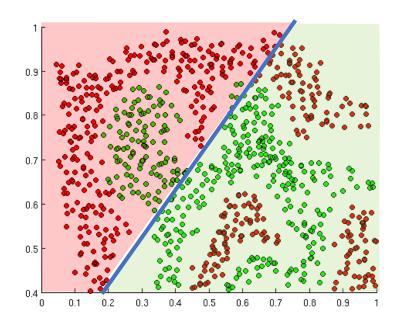


What if we wanted to build a Neural Network to distinguish green vs red points?



Importance of Activation Functions

The purpose of activation functions is to **introduce non-linearities** into the network

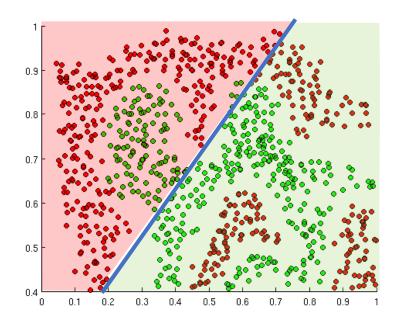


Linear Activation functions produce linear decisions no matter the network size

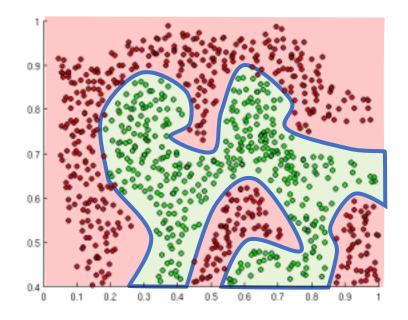


Importance of Activation Functions

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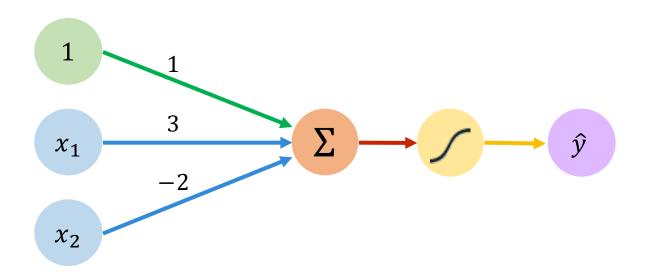


Linear Activation functions produce linear decisions no matter the network size



Non-linearities allow us to approximate arbitrarily complex functions





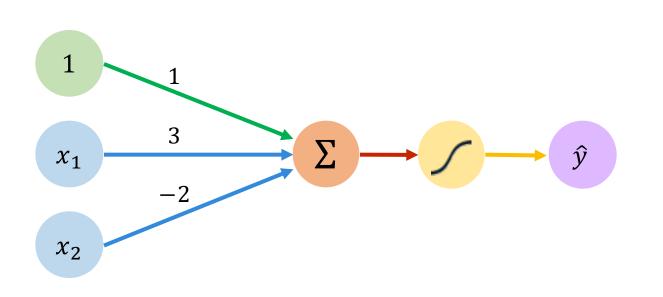
We have:
$$\theta_0 = 1$$
 and $\boldsymbol{\theta} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

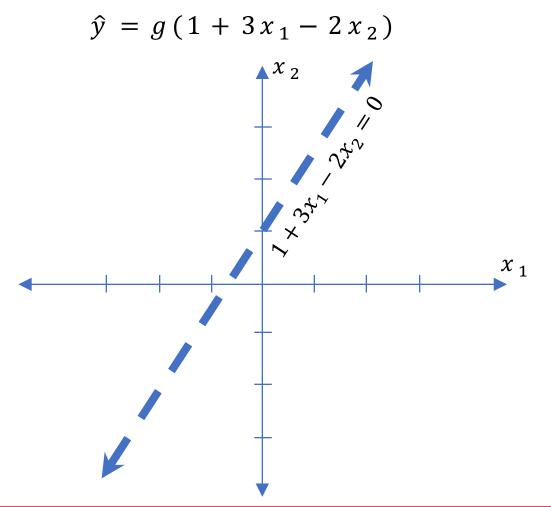
$$\hat{y} = g \left(\theta_0 + X^T \theta \right)$$

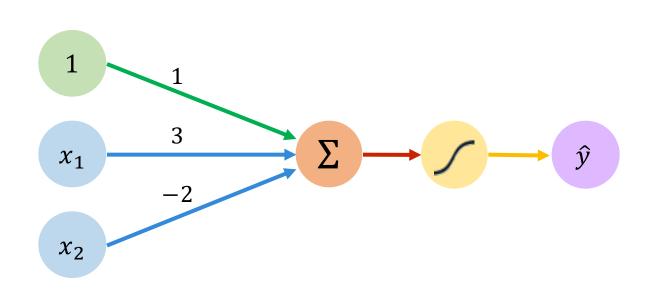
$$= g \left(1 + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} 3 \\ -2 \end{bmatrix} \right)$$

$$\hat{y} = g \left(1 + 3x_1 - 2x_2 \right)$$

This is just a line in 2D!



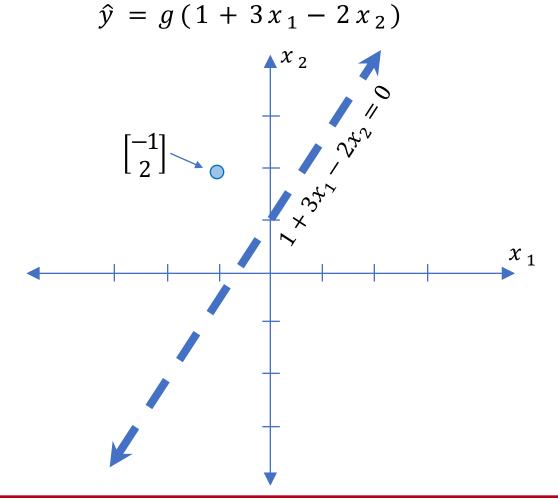


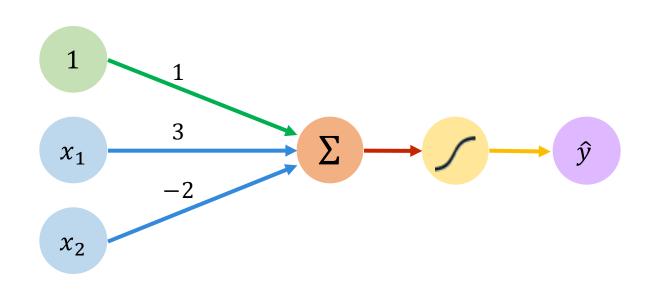


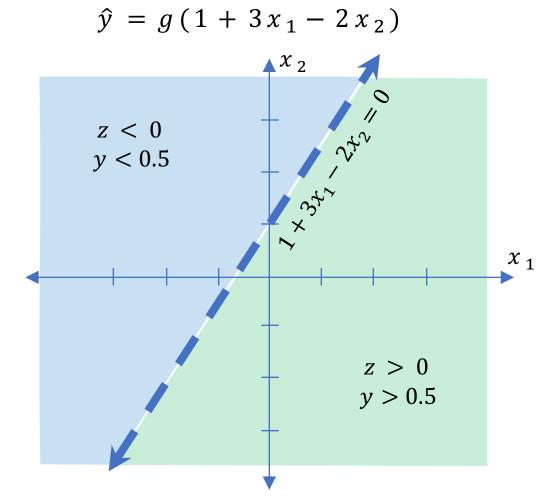
Assume we have input: $X = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$$\hat{y} = g(1 + (3*-1) - (2*2))$$

= $g(-6) \approx 0.002$

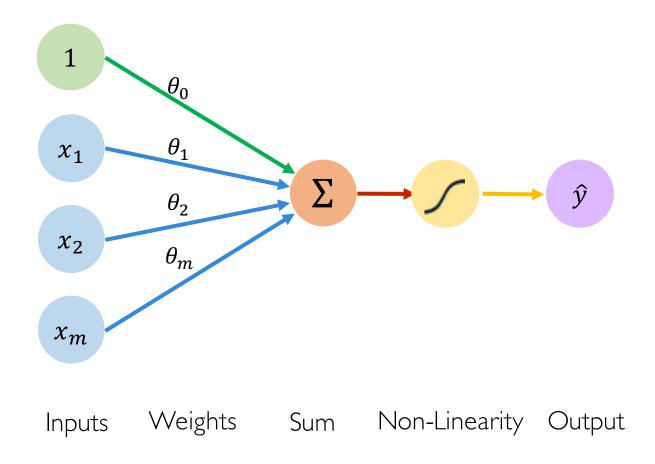






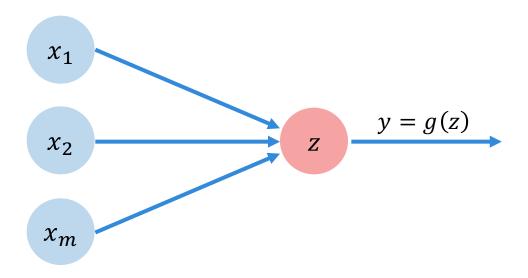
Building Neural Networks with Perceptrons

The Perceptron: Simplified



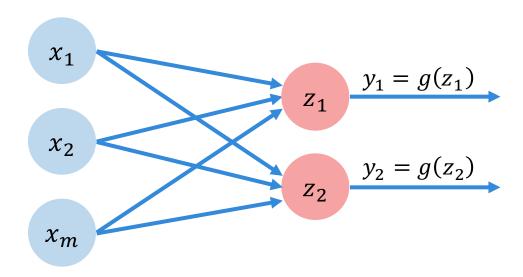


The Perceptron: Simplified



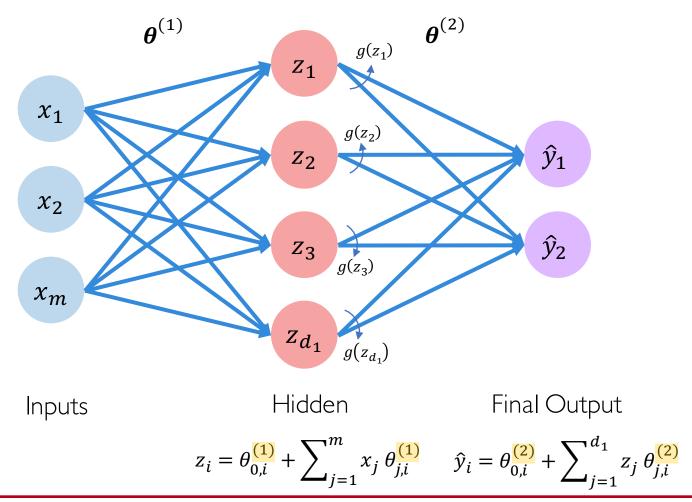
$$z = \theta_0 + \sum_{j=1}^m x_j \, \theta_j$$

Multi Output Perceptron



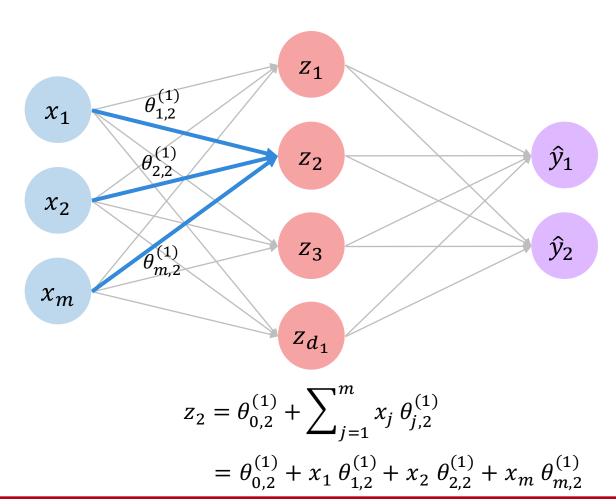
$$z_{\underline{i}} = \theta_{0,\underline{i}} + \sum_{j=1}^{m} x_j \ \theta_{j,\underline{i}}$$

Single Layer Neural Network

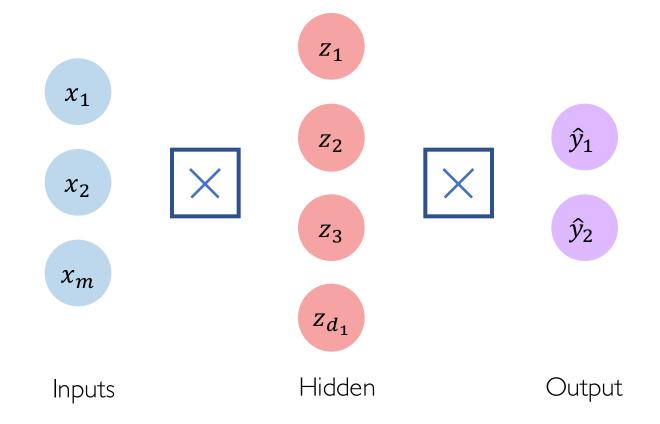




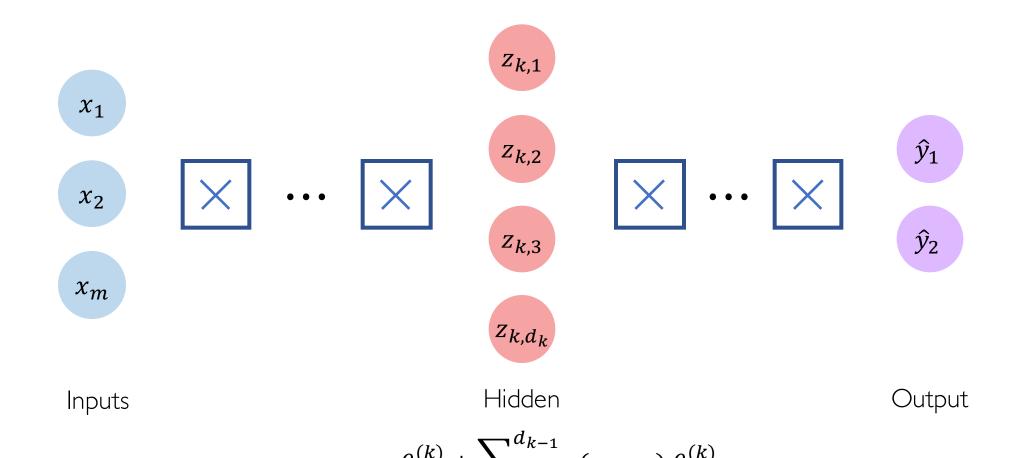
Single Layer Neural Network



Multi Output Perceptron



Deep Neural Network





Applying Neural Networks

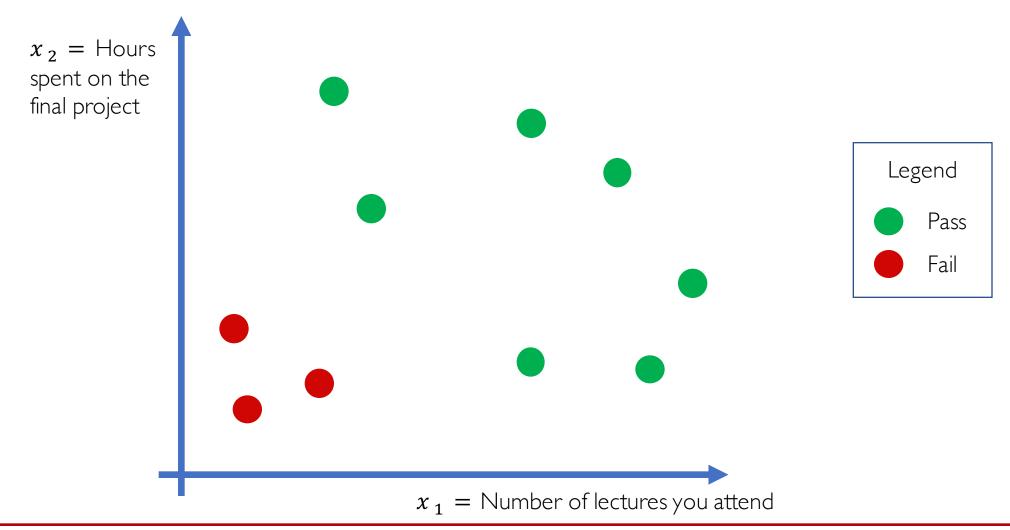
Example Problem

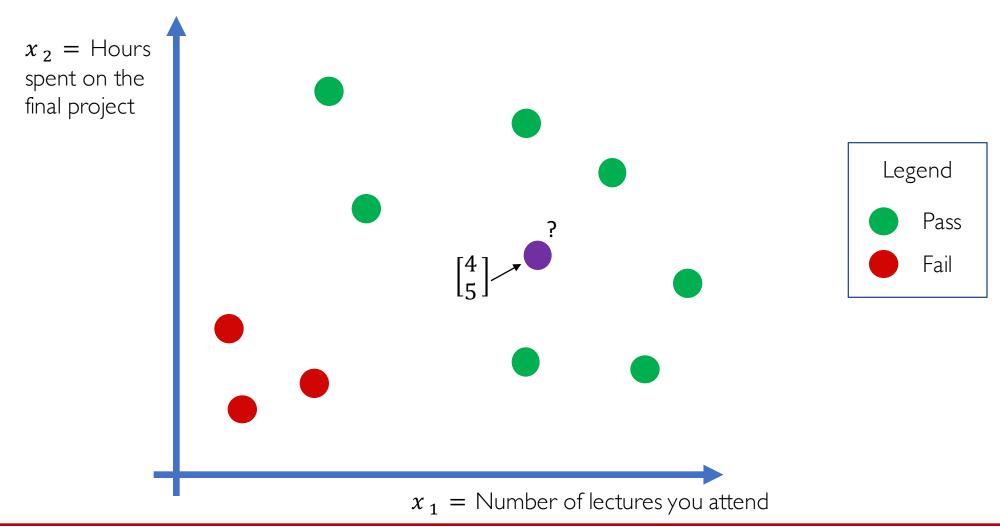
Will I pass this class?

Let's start with a simple two feature model

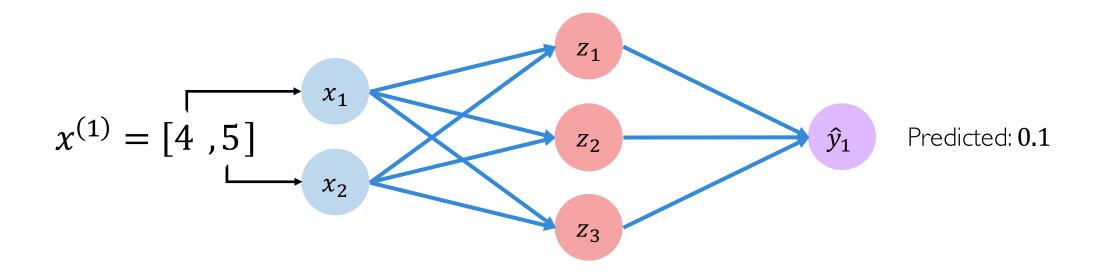
 x_1 = Number of lectures you attend

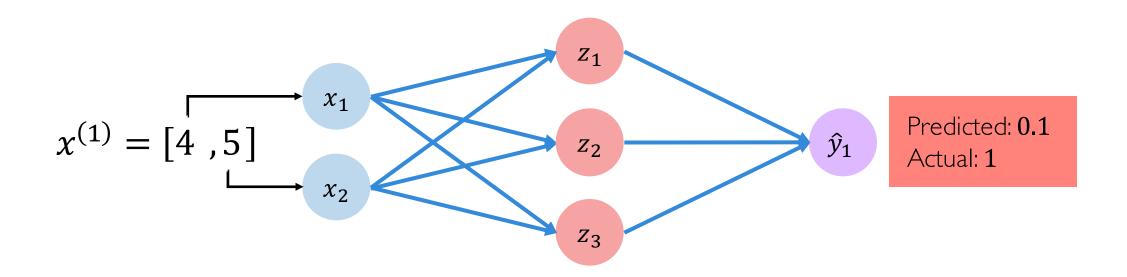
 x_2 = Hours spent on the final project





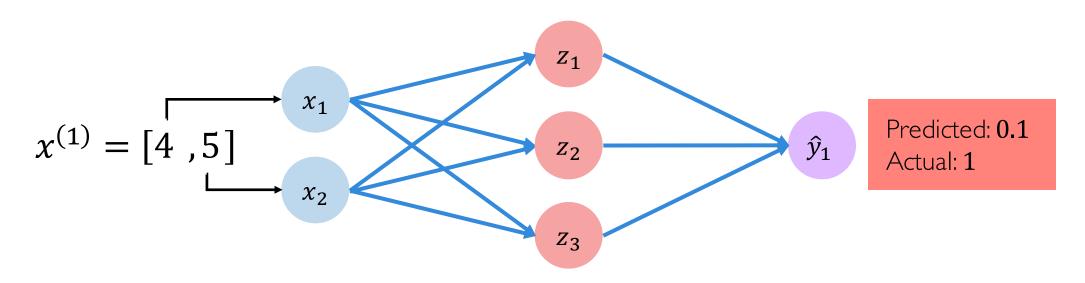






Quantifying Loss

The **loss** of our network measures the cost incurred from incorrect predictions



$$\mathcal{L}\left(f\left(x^{(i)};\boldsymbol{\theta}\right),y^{(i)}\right)$$
Predicted Actual

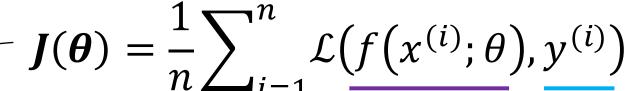
Empirical Loss

The **empirical loss** measures the total loss over our entire dataset

$$x = \begin{bmatrix} 4 & 5 \\ 2 & 1 \\ 5 & 8 \\ \vdots & \vdots \end{bmatrix} \qquad x_2 \qquad x_2 \qquad x_3 \qquad \begin{bmatrix} f(x) & y \\ 0 & 1 \\ 0 & 8 \\ 0 & 6 \\ \vdots \end{bmatrix} \qquad \begin{bmatrix} f(x) & y \\ 0 & 1 \\ 0 & 0 \\ \vdots \end{bmatrix}$$

Also known as:

- Objective function
- Cost function
- Empirical Risk

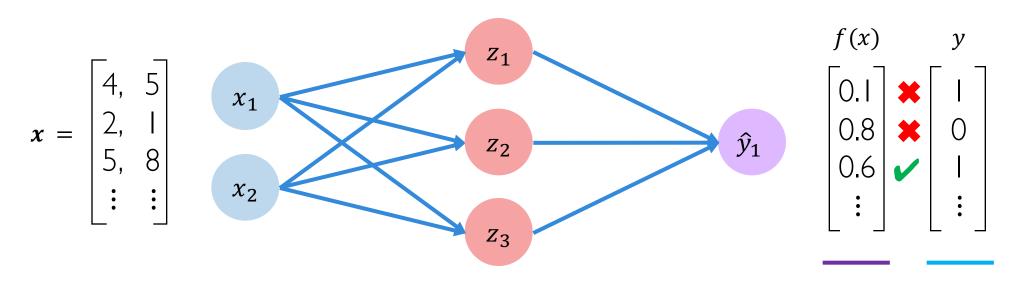


Predicted

Actual

Binary Cross Entropy Loss

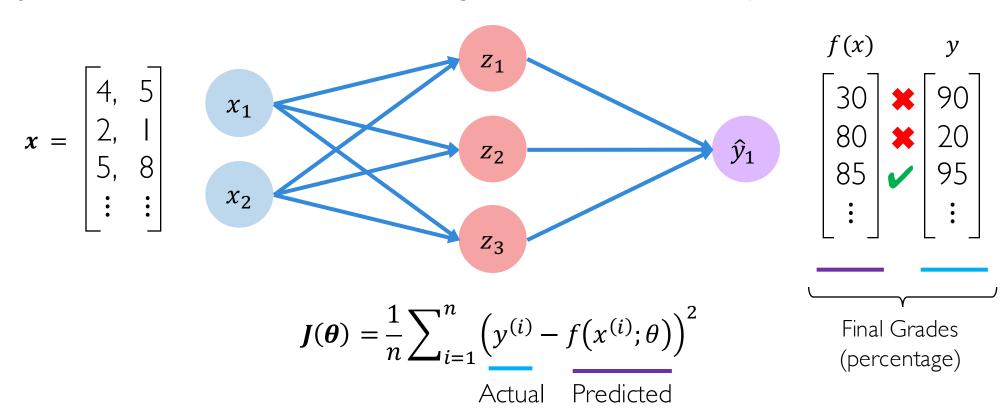
Cross entropy loss can be used with models that output a probability between 0 and 1



$$J(\theta) = \frac{1}{n} \sum_{i=1}^{n} y^{(i)} \log \left(f(x^{(i)}; \theta) \right) + (1 - y^{(i)}) \log \left(1 - f(x^{(i)}; \theta) \right)$$
Actual Predicted Actual Predicted

Mean Squared Error Loss

Mean squared error loss can be used with regression models that output continuous real numbers



Training Neural Networks

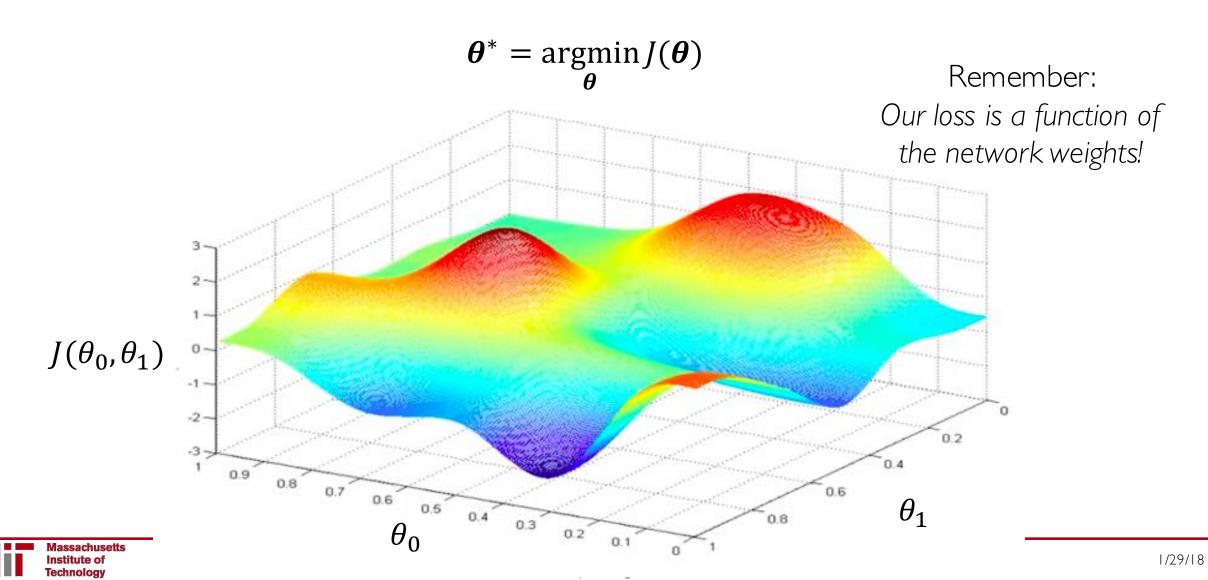
We want to find the network weights that achieve the lowest loss

$$\boldsymbol{\theta}^* = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \frac{1}{n} \sum_{i=1}^n \mathcal{L}(f(x^{(i)}; \boldsymbol{\theta}), y^{(i)})$$
$$\boldsymbol{\theta}^* = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} J(\boldsymbol{\theta})$$

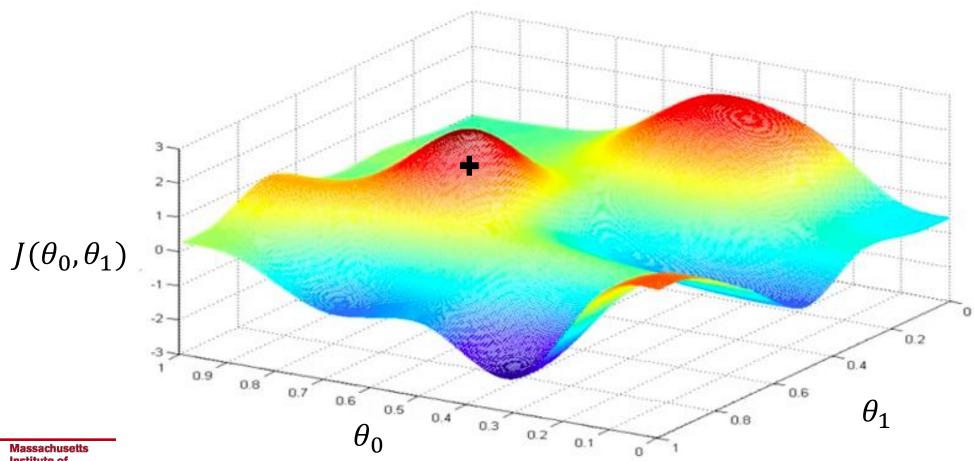
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$$\boldsymbol{\theta}^* = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} J(\boldsymbol{\theta})$$
Remember:
$$\boldsymbol{\theta} = \{\theta^{(0)}, \theta^{(1)}, \dots\}$$

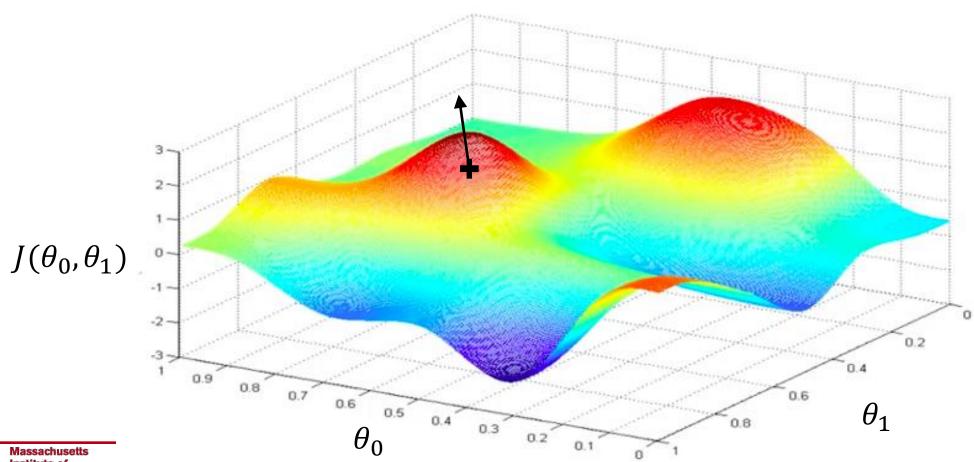


Randomly pick an initial (θ_0, θ_1)



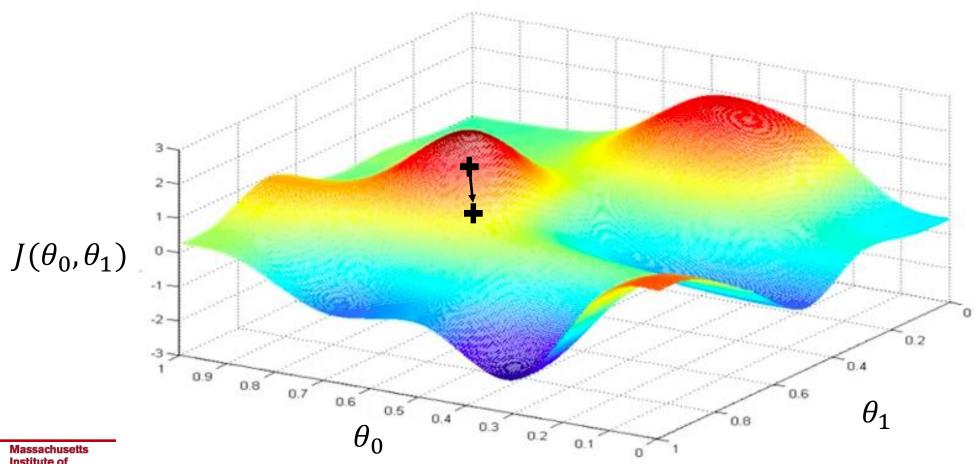








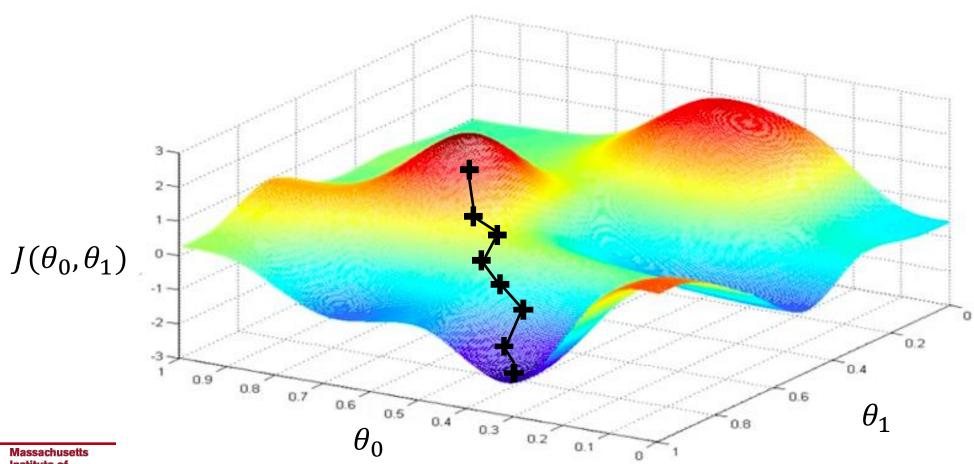
Take small step in opposite direction of gradient





Gradient Descent

Repeat until convergence





Gradient Descent

Algorithm

I. Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$

💎 weights = tf.random_normal(shape, stddev=sigma)

- 2. Loop until convergence:
- 3. Compute gradient, $\frac{\partial J(\theta)}{\partial \theta}$
- 4. Update weights, $\theta \leftarrow \theta \eta \frac{\partial J(\theta)}{\partial \theta}$
- 5. Return weights

```
grads = tf.gradients(ys=loss, xs=weights)
```

Gradient Descent

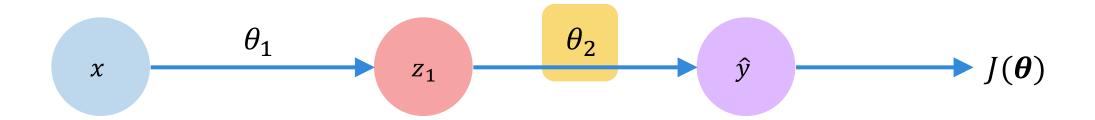
Algorithm

Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$

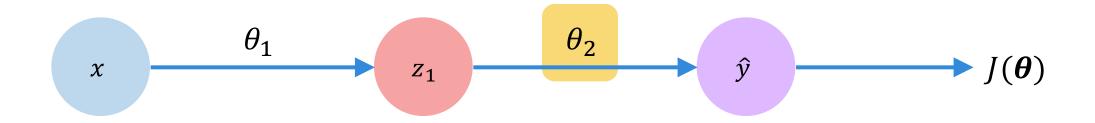
weights = tf.random_normal(shape, stddev=sigma)

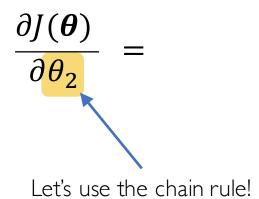
- Loop until convergence:
- 3.
- Compute gradient, $\frac{\partial J(\theta)}{\partial \theta}$ Update weights, $\theta \leftarrow \theta \eta \frac{\partial J(\theta)}{\partial \theta}$
- 5. Return weights

```
= tf.gradients(ys=loss, xs=weights)
```

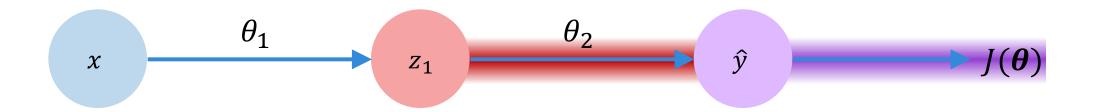


How does a small change in one weight (ex. θ_2) affect the final loss $J(\theta)$?

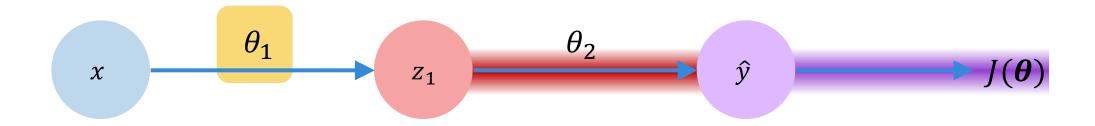




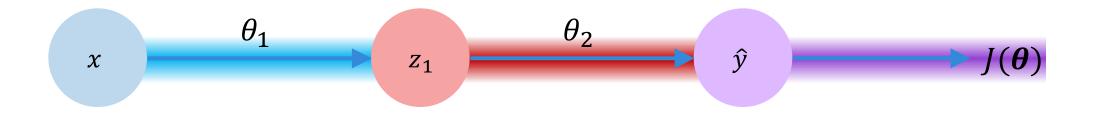




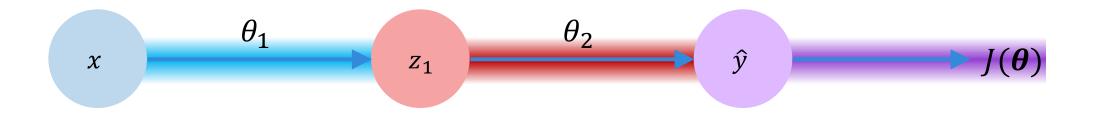
$$\frac{\partial J(\boldsymbol{\theta})}{\partial \theta_2} = \frac{\partial J(\boldsymbol{\theta})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial \theta_2}$$



$$\frac{\partial J(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}_1} = \frac{\partial J(\boldsymbol{\theta})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial \boldsymbol{\theta}_1}$$
Apply chain rule! Apply chain rule!



$$\frac{\partial J(\boldsymbol{\theta})}{\partial \theta_1} = \frac{\partial J(\boldsymbol{\theta})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial z_1} * \frac{\partial z_1}{\partial \theta_1}$$

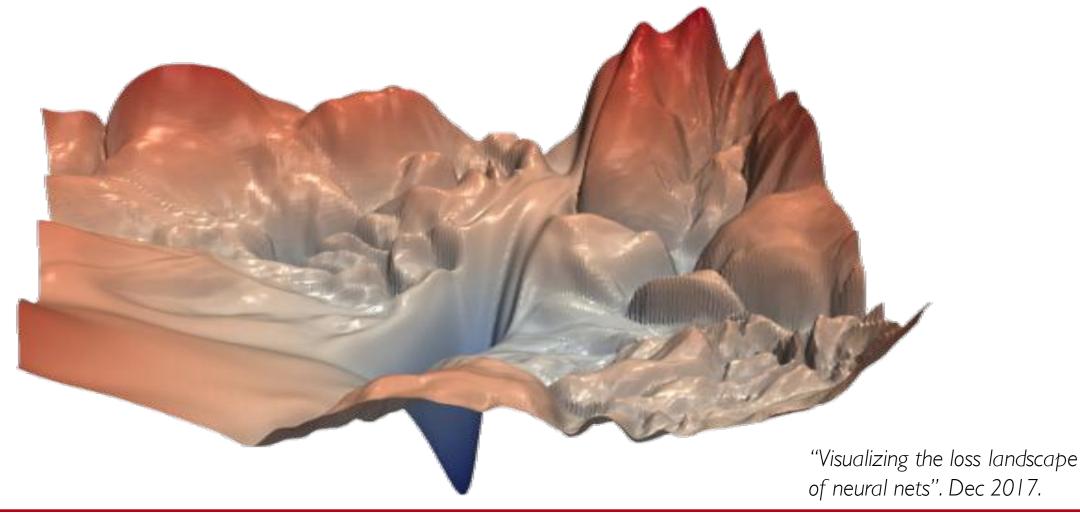


$$\frac{\partial J(\boldsymbol{\theta})}{\partial \theta_1} = \frac{\partial J(\boldsymbol{\theta})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial z_1} * \frac{\partial z_1}{\partial \theta_1}$$

Repeat this for every weight in the network using gradients from later layers

Neural Networks in Practice: Optimization

Training Neural Networks is Difficult



Loss Functions Can Be Difficult to Optimize

Remember:

Optimization through gradient descent

$$\theta \leftarrow \theta - \eta \, \frac{\partial J(\theta)}{\partial \theta}$$

Loss Functions Can Be Difficult to Optimize

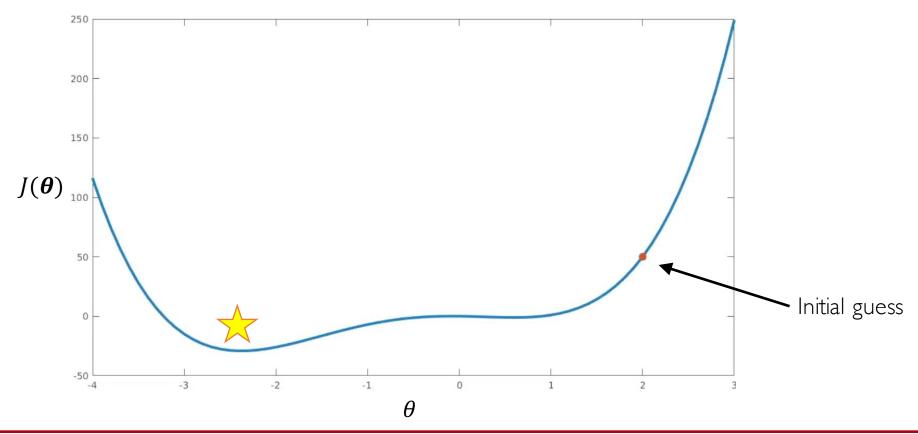
Remember:

Optimization through gradient descent

$$\theta \leftarrow \theta - \eta \frac{\partial J(\theta)}{\partial \theta}$$
How can we set the learning rate?

Setting the Learning Rate

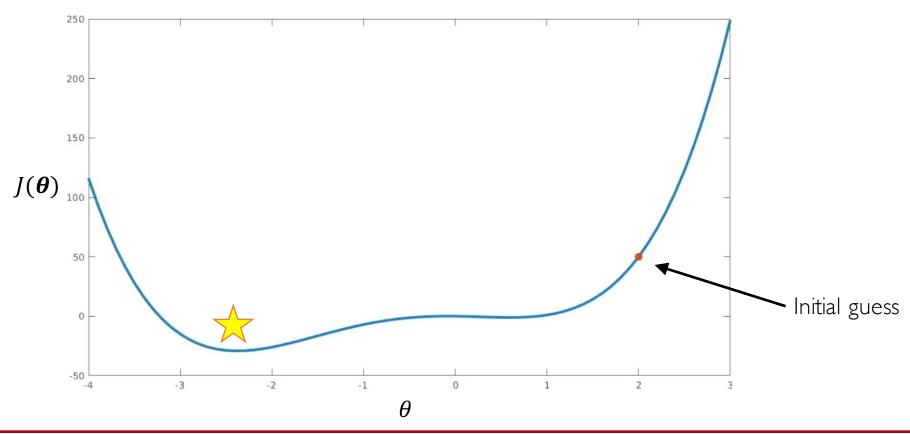
Small learning rate converges slowly and gets stuck in false local minima





Setting the Learning Rate

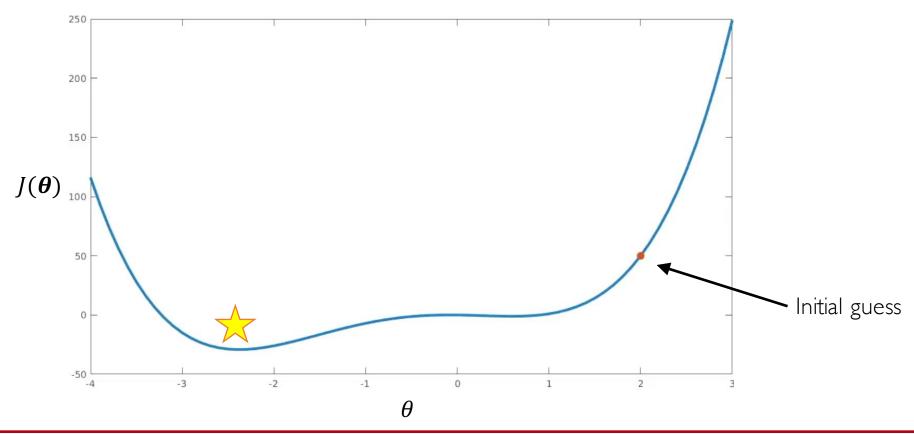
Large learning rates overshoot, become unstable and diverge





Setting the Learning Rate

Stable learning rates converge smoothly and avoid local minima





How to deal with this?

Idea I:

Try lots of different learning rates and see what works "just right"

How to deal with this?

Idea I:

Try lots of different learning rates and see what works "just right"

Idea 2:

Do something smarter!

Design an adaptive learning rate that "adapts" to the landscape

Adaptive Learning Rates

- Learning rates are no longer fixed
- Can be made larger or smaller depending on:
 - how large gradient is
 - how fast learning is happening
 - size of particular weights
 - etc...

Adaptive Learning Rate Algorithms

- Momentum
- Adagrad
- Adadelta
- Adam
- RMSProp











Qian et al. "On the momentum term in gradient descent learning algorithms." 1999.

Duchi et al. "Adaptive Subgradient Methods for Online Learning and Stochastic Optimization." 2011.

Zeiler et al. "ADADELTA: An Adaptive Learning Rate Method." 2012.

Kingma et al. "Adam: A Method for Stochastic Optimization." 2014.

Additional details: http://ruder.io/optimizing-gradient-descent/

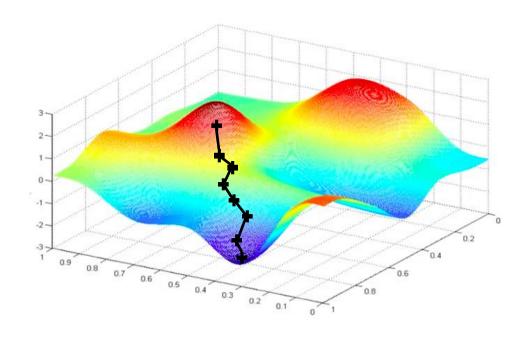


Neural Networks in Practice: Mini-batches

Gradient Descent

Algorithm

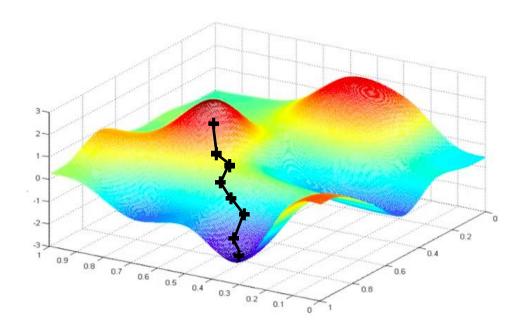
- I. Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
- 2. Loop until convergence:
- 3. Compute gradient, $\frac{\partial J(\theta)}{\partial \theta}$
- 4. Update weights, $\theta \leftarrow \theta \eta \frac{\partial J(\theta)}{\partial \theta}$
- 5. Return weights



Gradient Descent

Algorithm

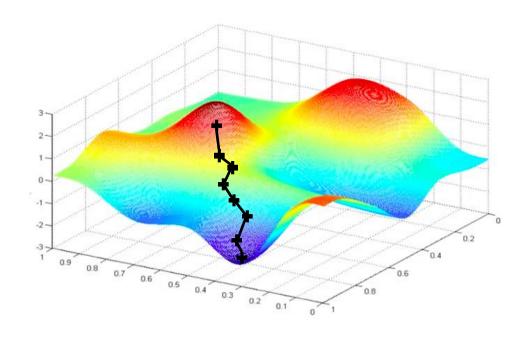
- Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
- Loop until convergence:
- Compute gradient, $\frac{\partial J(\theta)}{\partial \theta}$ Update weights, $\theta \leftarrow \theta \eta \frac{\partial J(\theta)}{\partial \theta}$ 3.
- 5. Return weights



Can be very computational to compute!

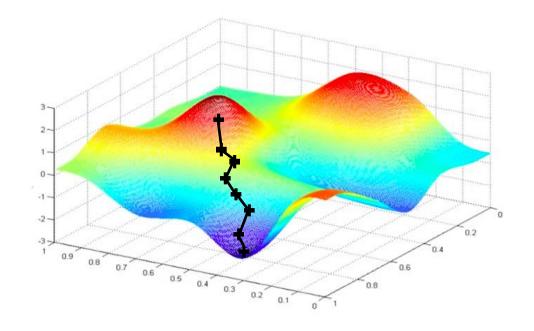
Algorithm

- I. Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
- 2. Loop until convergence:
- 3. Pick single data point i
- 4. Compute gradient, $\frac{\partial J_i(\theta)}{\partial \theta}$
- 5. Update weights, $\theta \leftarrow \theta \eta \frac{\partial J(\theta)}{\partial \theta}$
- 6. Return weights



Algorithm

- I. Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
- 2. Loop until convergence:
- 3. Pick single data point *i*
- 4. Compute gradient, $\frac{\partial J_i(\theta)}{\partial \theta}$
- 5. Update weights, $\theta \leftarrow \theta \eta \frac{\partial J(\theta)}{\partial \theta}$
- 6. Return weights



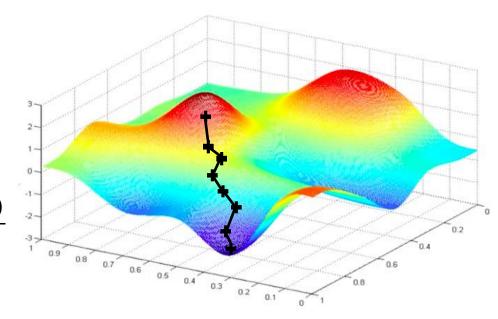
Easy to compute but **very noisy** (stochastic)!

Algorithm

- I. Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
- 2. Loop until convergence:
- 3. Pick batch of B data points

4. Compute gradient,
$$\frac{\partial J(\theta)}{\partial \theta} = \frac{1}{B} \sum_{k=1}^{B} \frac{\partial J_k(\theta)}{\partial \theta}$$

- 5. Update weights, $\theta \leftarrow \theta \eta \frac{\partial J(\theta)}{\partial \theta}$
- 6. Return weights

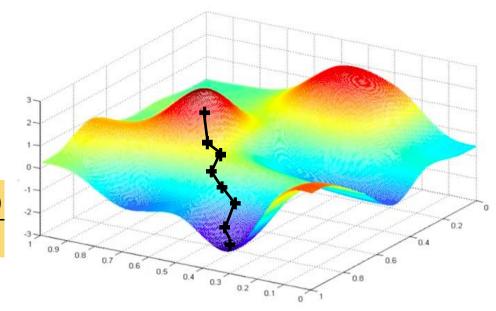


Algorithm

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Fast to compute and a much better estimate of the true gradient!

Mini-batches while training

More accurate estimation of gradient

Smoother convergence Allows for larger learning rates



Mini-batches while training

More accurate estimation of gradient

Smoother convergence
Allows for larger learning rates

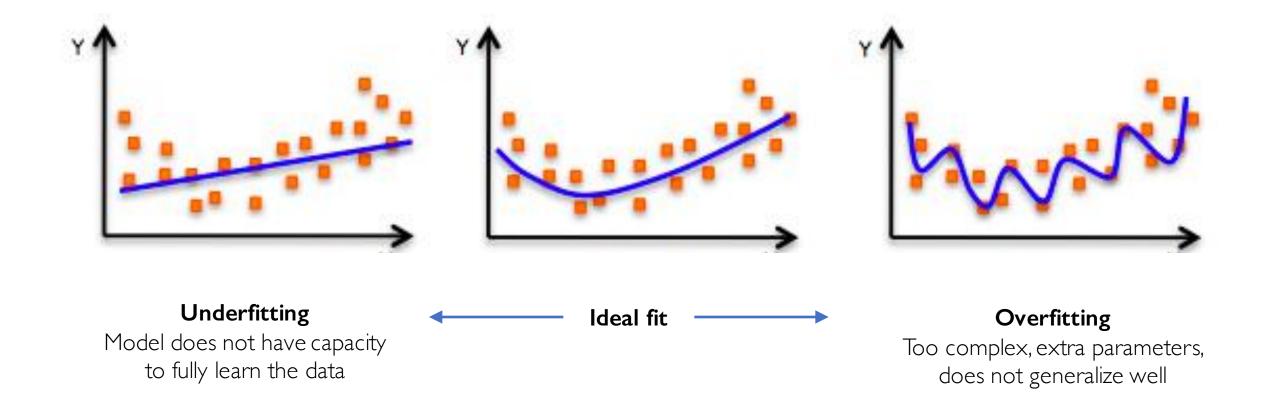
Mini-batches lead to fast training!

Can parallelize computation + achieve significant speed increases on GPU's



Neural Networks in Practice: Overfitting

The Problem of Overfitting



Regularization

What is it?

Technique that constrains our optimization problem to discourage complex models

Regularization

What is it?

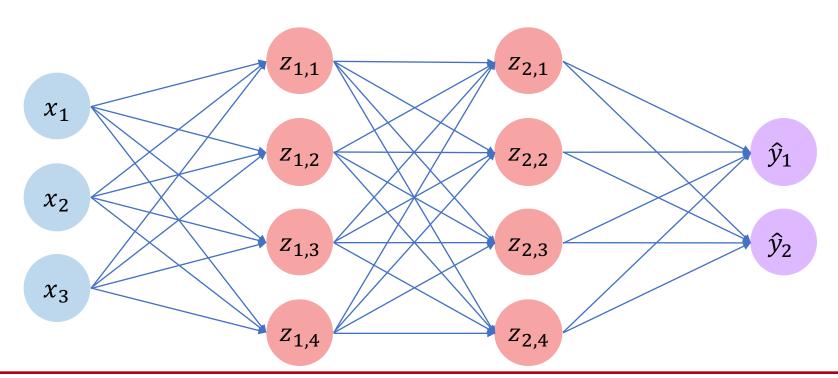
Technique that constrains our optimization problem to discourage complex models

Why do we need it?

Improve generalization of our model on unseen data

Regularization I: Dropout

During training, randomly set some activations to 0

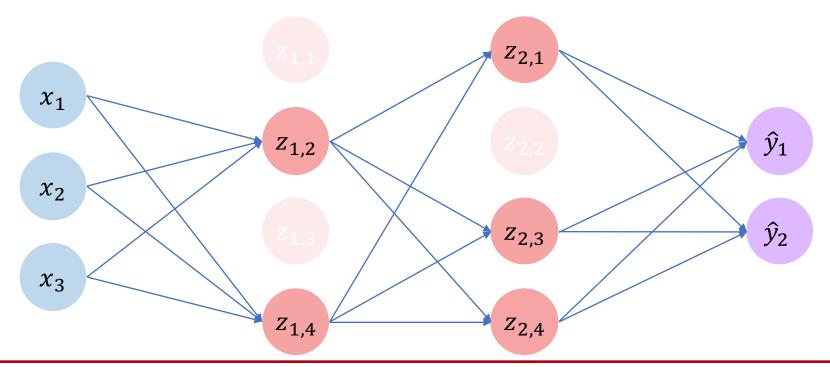




Regularization I: Dropout

- During training, randomly set some activations to 0
 - Typically 'drop' 50% of activations in layer
 - Forces network to not rely on any I node



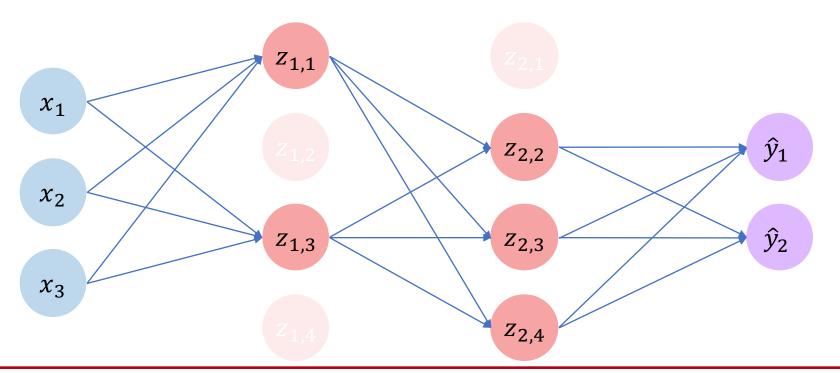




Regularization I: Dropout

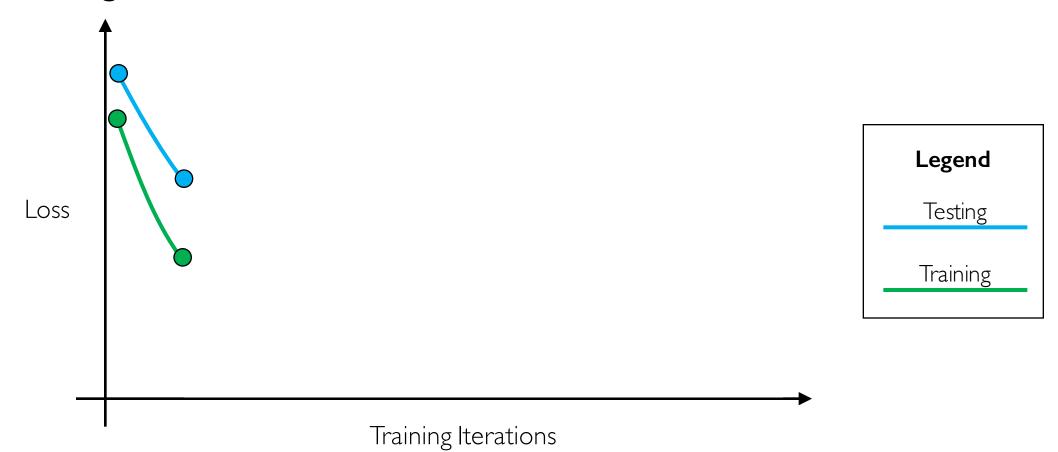
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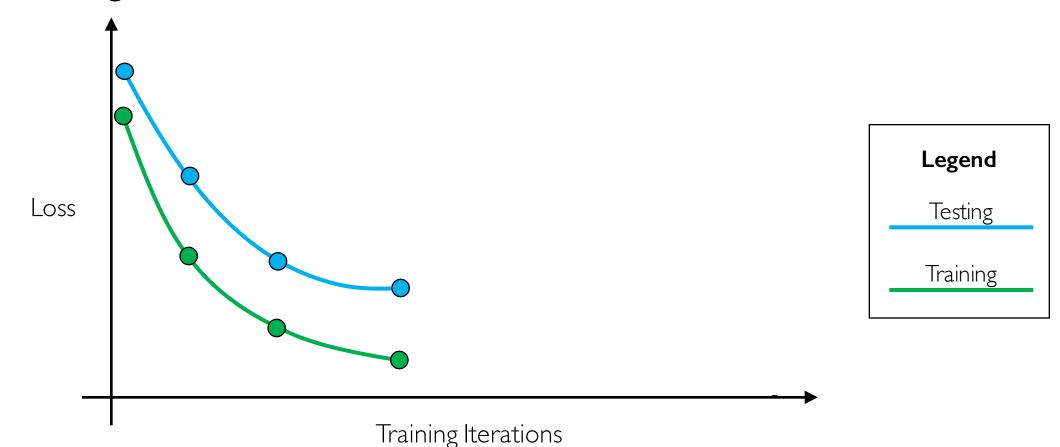


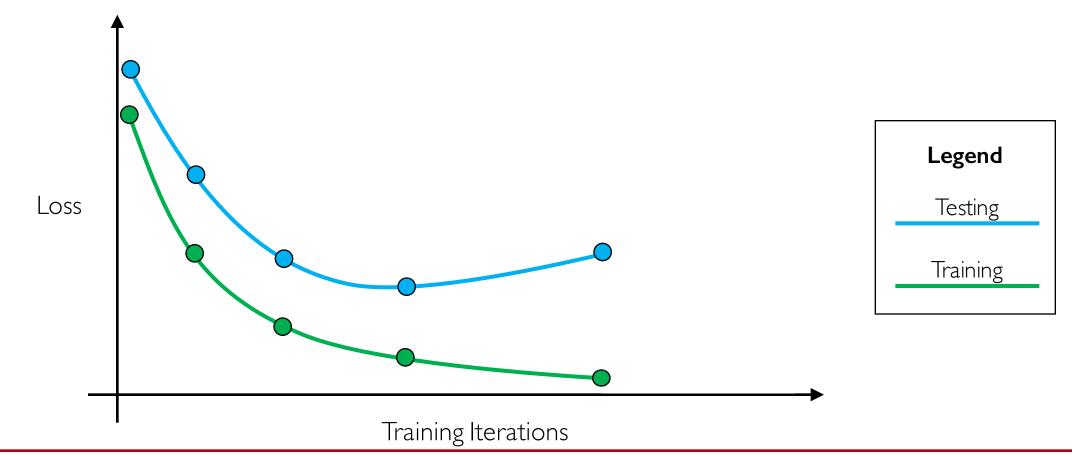




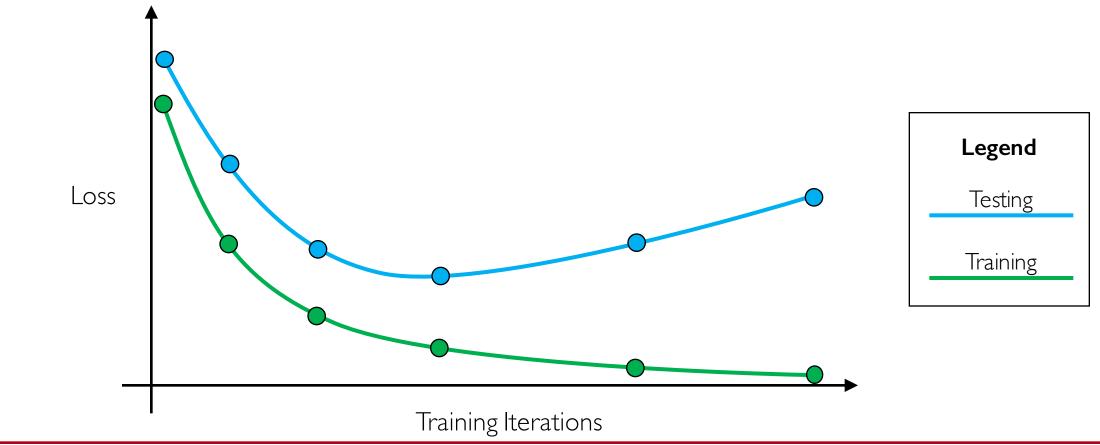




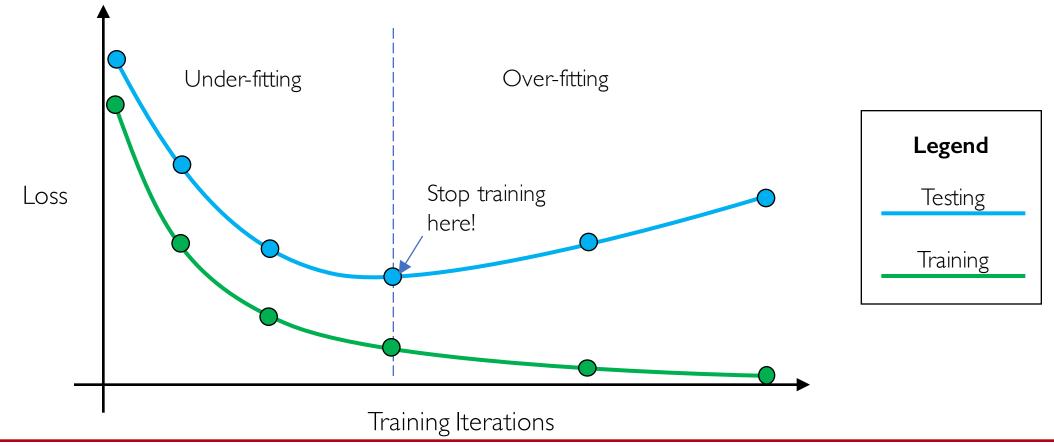








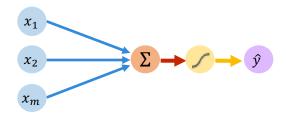




Core Foundation Review

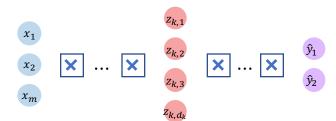
The Perceptron

- Structural building blocks
- Nonlinear activation functions



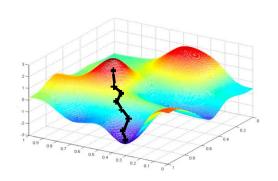
Neural Networks

- Stacking Perceptrons to form neural networks
- Optimization through backpropagation



Training in Practice

- Adaptive learning
- Batching
- Regularization



Questions?