Deep Reinforcement Learning
MIT 6.S191

Alexander Amini

January 30, 2019

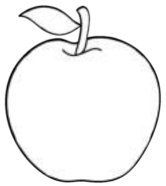
Classes of Learning Problems

Supervised Learning

Data: (x, y)
x is data, y is label

Goal: Learn function to map
$$x \rightarrow y$$

Apple example:



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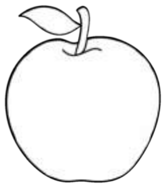

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Unsupervised Learning

Data: x
x is data, no labels!

Goal: Learn underlying structure

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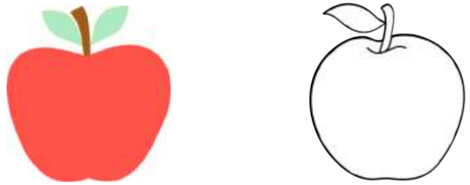

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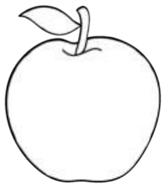

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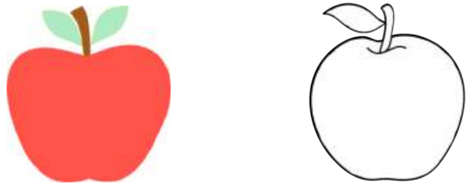

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Reinforcement Learning

Data: state-action pairs

Goal: Maximize future rewards over many time steps

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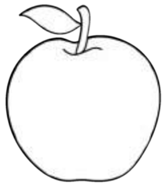

Eat this thing because it will keep you alive.

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This thing is like the other thing.

RL: our focus today.

Reinforcement Learning

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Reinforcement Learning (RL): Key Concepts

AGENT

Agent: takes actions.

Reinforcement Learning (RL): Key Concepts

AGENT

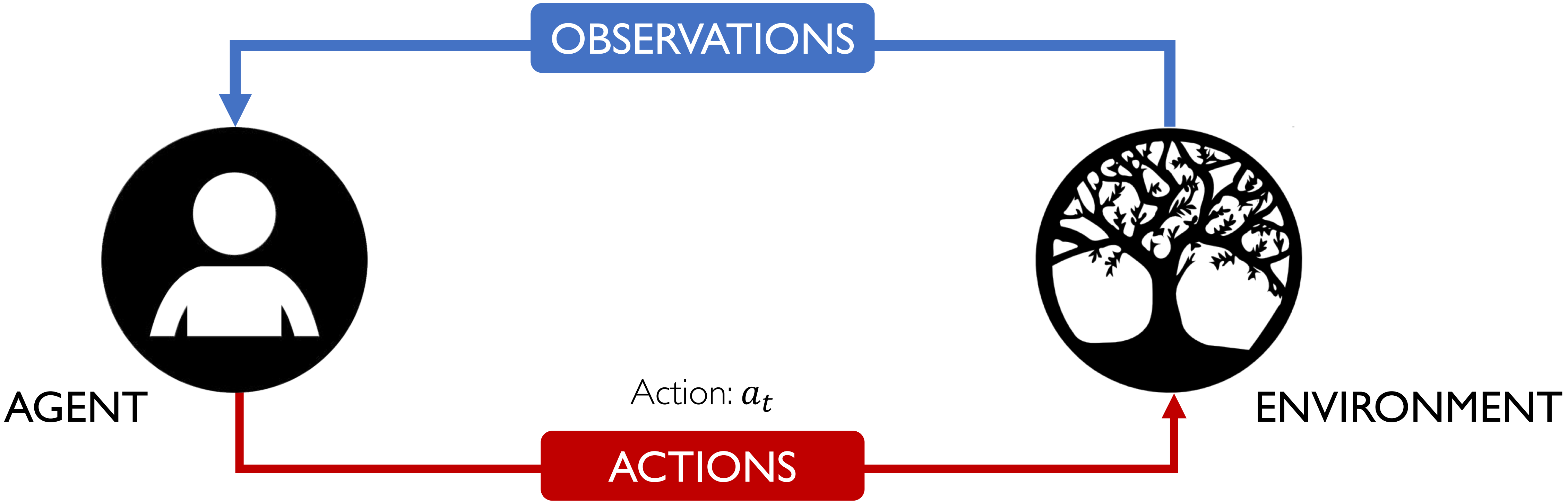
ENVIRONMENT

Environment: the world in which the agent exists and operates.

Reinforcement Learning (RL): Key Concepts

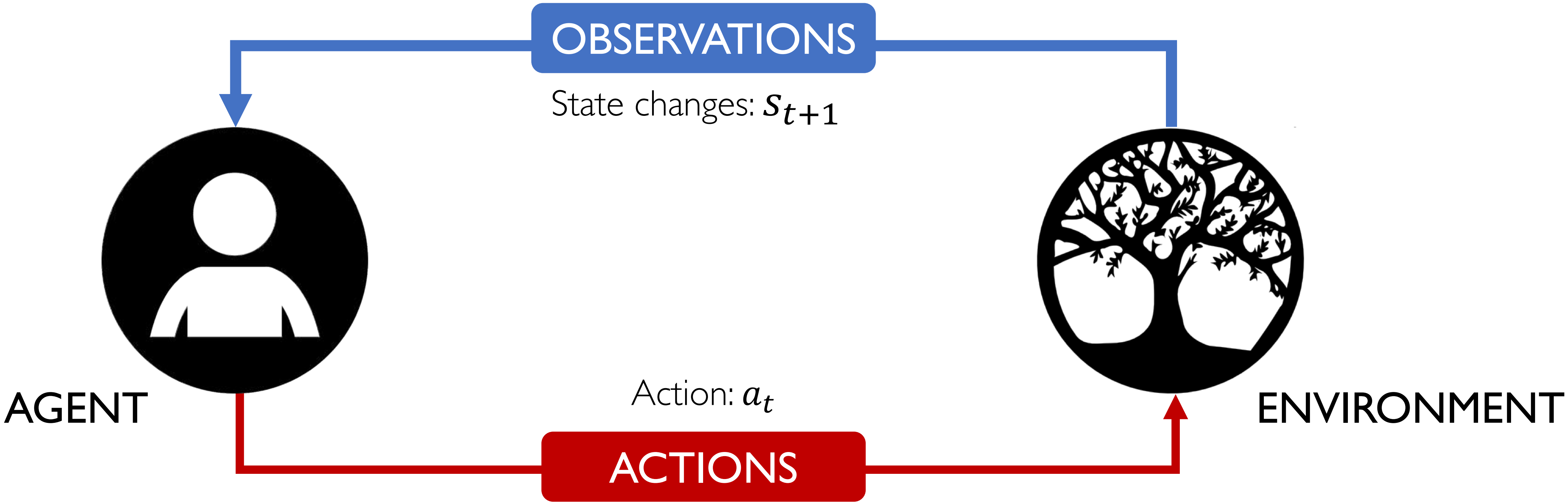
Action: a move the agent can make in the environment.

Reinforcement Learning (RL): Key Concepts



Observations: of the environment after taking actions.

Reinforcement Learning (RL): Key Concepts



State: a situation which the agent perceives.

Reinforcement Learning (RL): Key Concepts

Reward: feedback that measures the success or failure of the agent's action.

Reinforcement Learning (RL): Key Concepts

Total Reward

$$R_t = \sum_{i=t}^{\infty} r_i$$

Reinforcement Learning (RL): Key Concepts

Total Reward

$$R_t = \sum_{i=t}^{\infty} r_i = r_t + r_{t+1} \ldots + r_{t+n} + \cdots$$

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Discounted Total Reward

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γ: discount factor

Defining the Q-function

$$R_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots$$

Total reward, R_t, is the discounted sum of all rewards obtained from time t

$$Q(s, a) = \mathbb{E}[R_t]$$

The Q-function captures the **expected total future reward** an agent in state, s, can receive by executing a certain action, a

How to take actions given a Q-function?

$$Q(s, a) = \mathbb{E}[R_t]$$

(state, action)

Ultimately, the agent needs a **policy $\boldsymbol{\pi(s)}$**, to infer the **best action to take** at its state, s

Strategy: the policy should choose an action that maximizes future reward

$$\pi^*(s) = \underset{a}{\operatorname{argmax}}\, Q(s, a)$$

Deep Reinforcement Learning Algorithms

Value Learning

Find $Q(s, a)$

$$a = \underset{a}{\operatorname{argmax}}\, Q(s, a)$$

Policy Learning

Find $\pi(s)$

Sample $a \sim \pi(s)$

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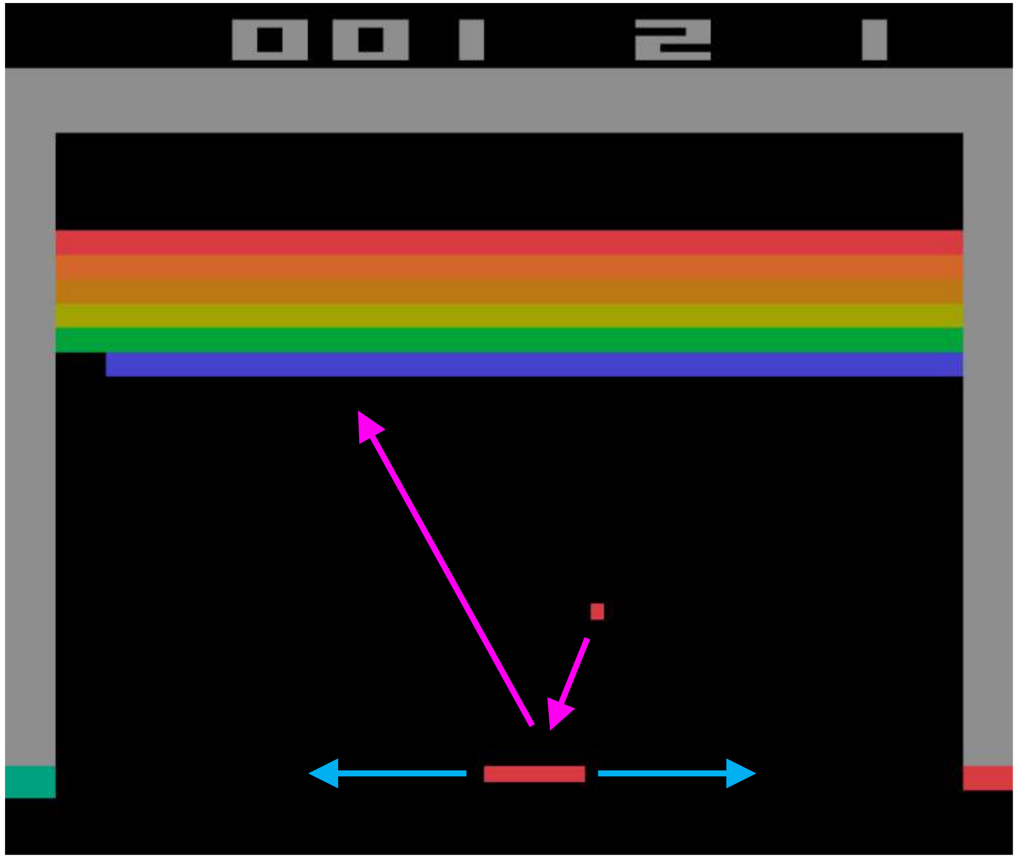
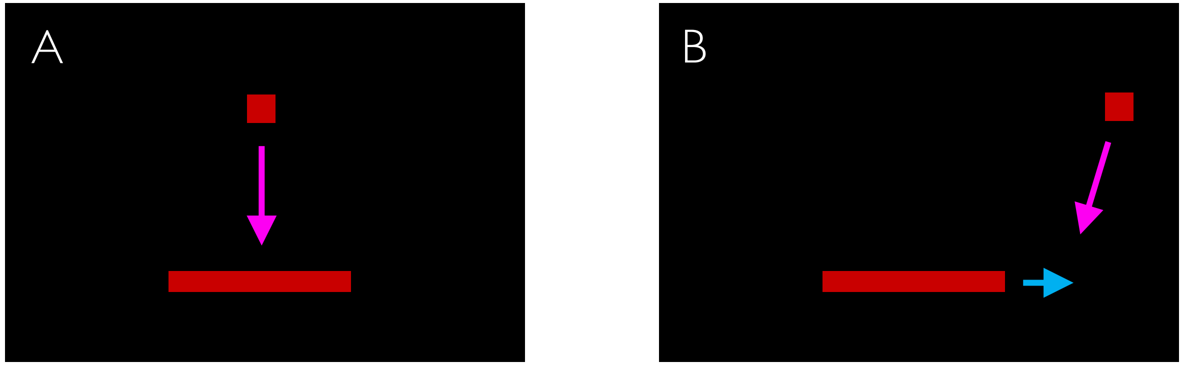
Digging deeper into the Q-function

Example: Atari Breakout



It can be very difficult for humans to accurately estimate Q-values



Which (s, a) pair has a higher Q-value?

Digging deeper into the Q-function

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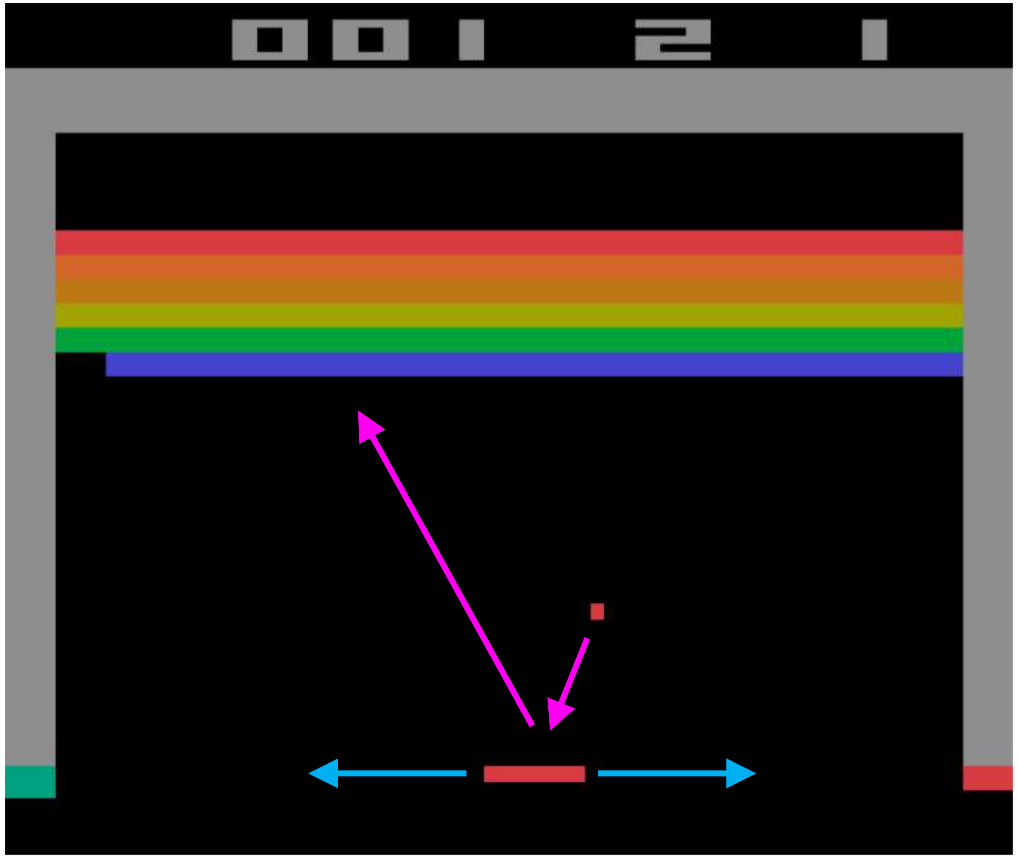

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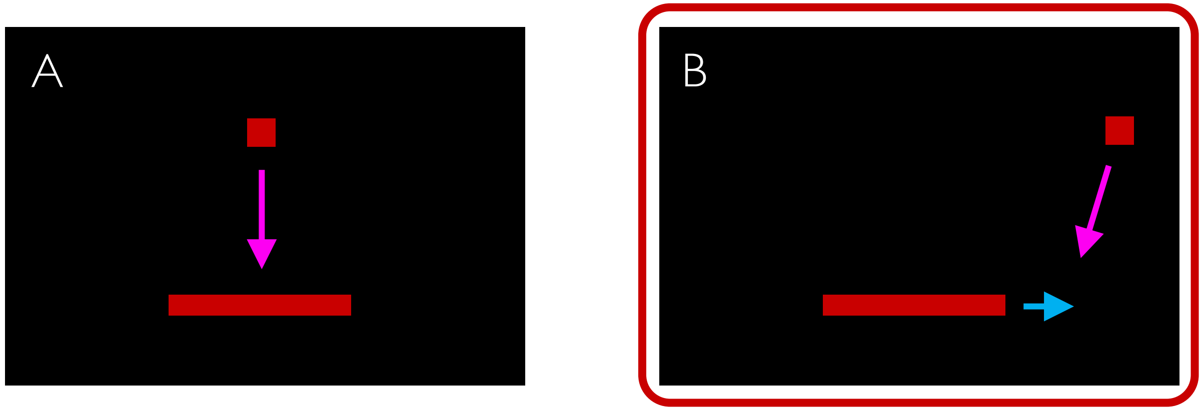
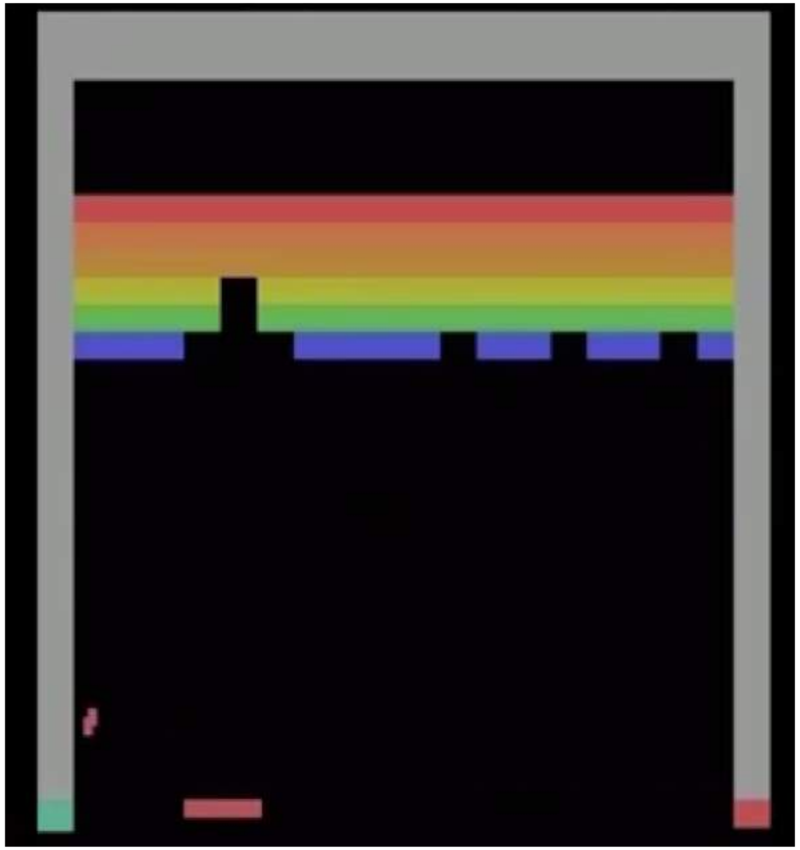


Which (s, a) pair has a higher Q-value?

Digging deeper into the Q-function

Example: Atari Breakout - Middle



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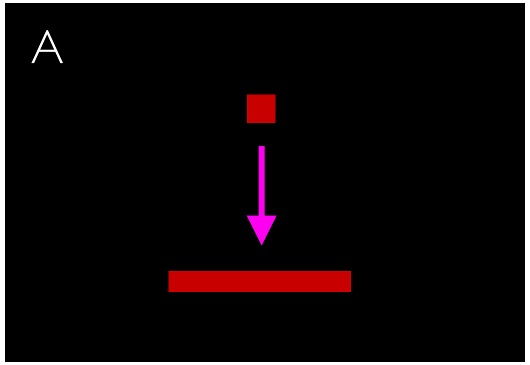

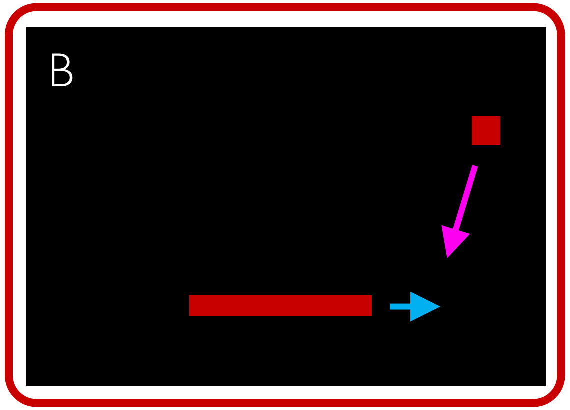

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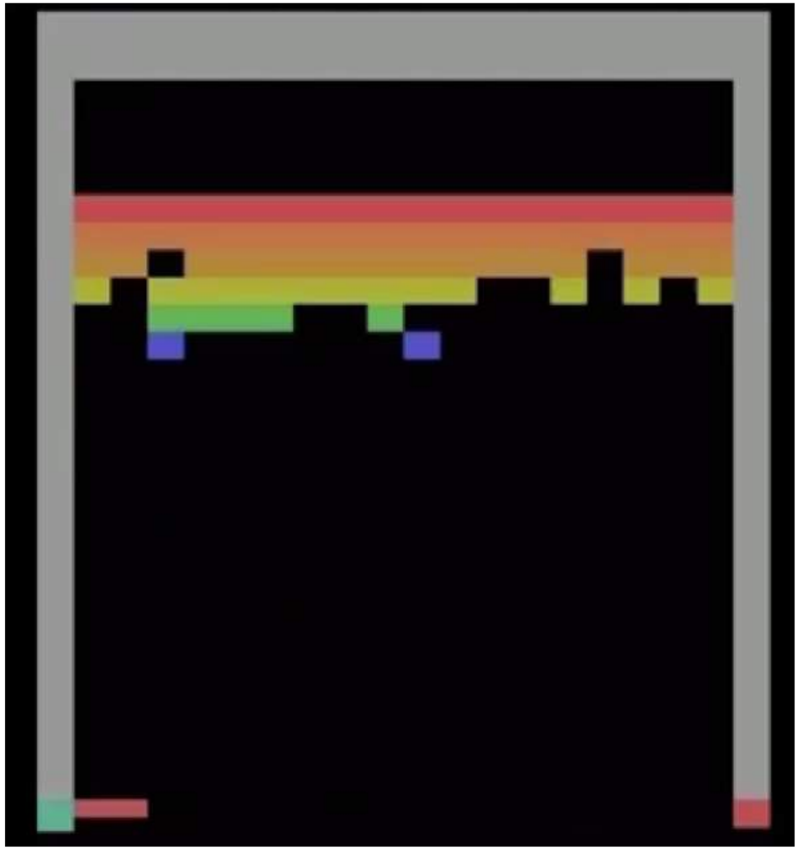
Digging deeper into the Q-function

Example: Atari Breakout - Side



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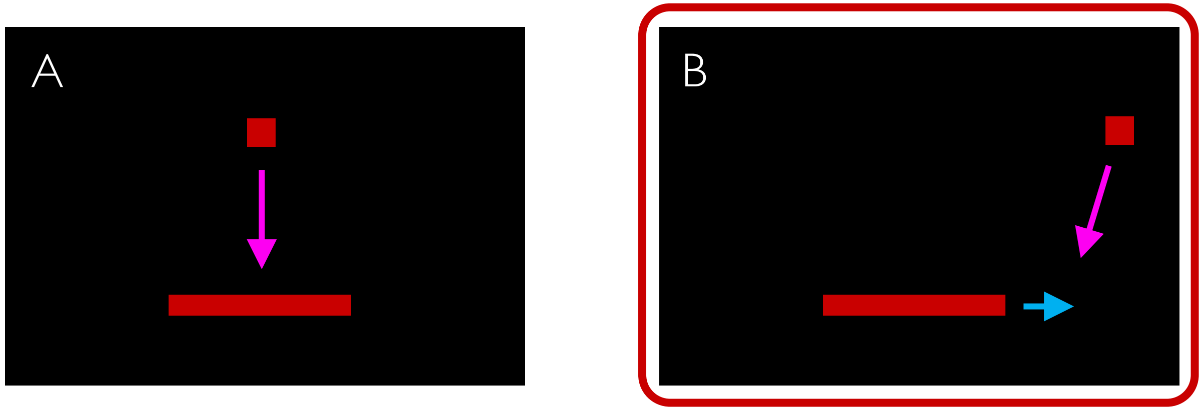

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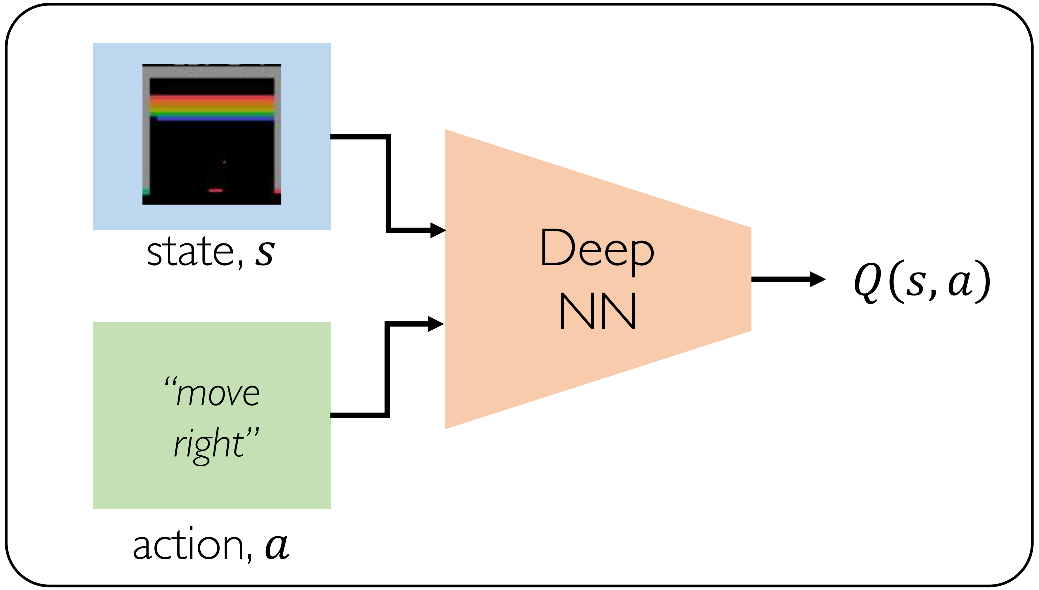
Deep Q Networks (DQN)

How can we use deep neural networks to model Q-functions?



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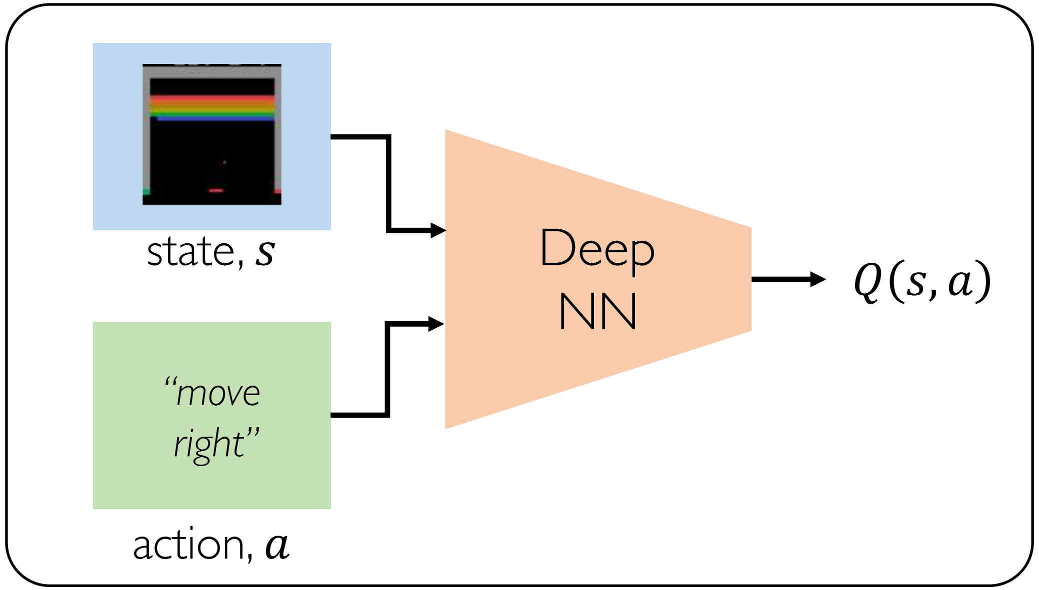

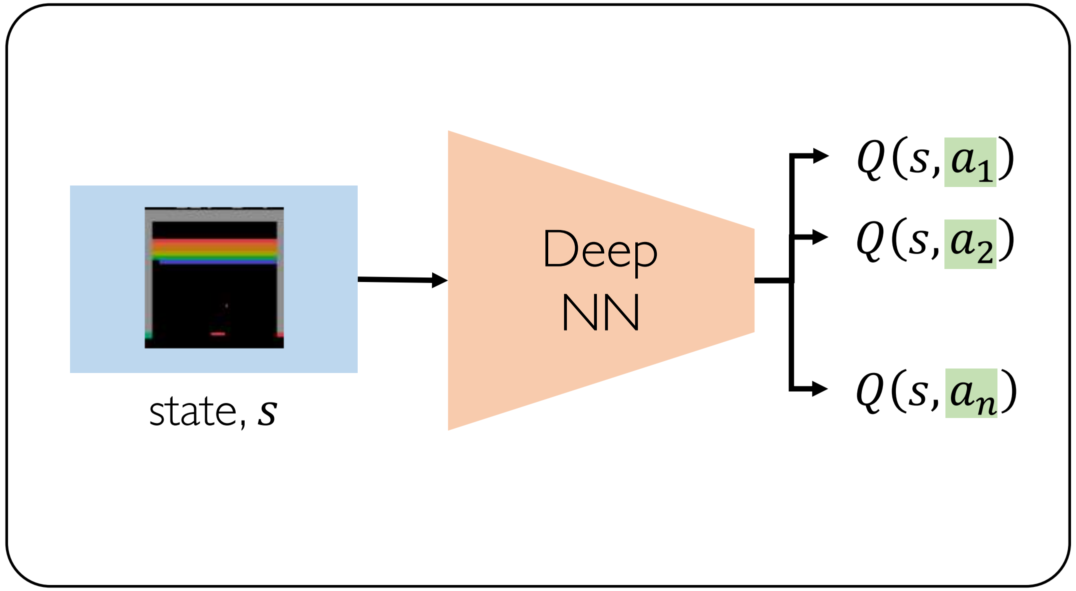

Deep Q Networks (DQN): Training

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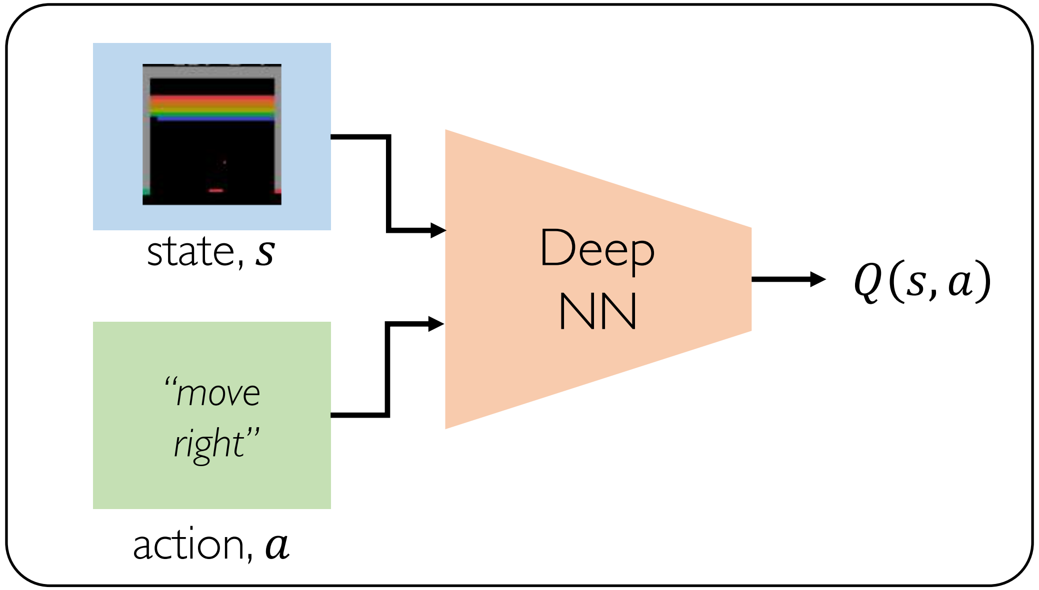

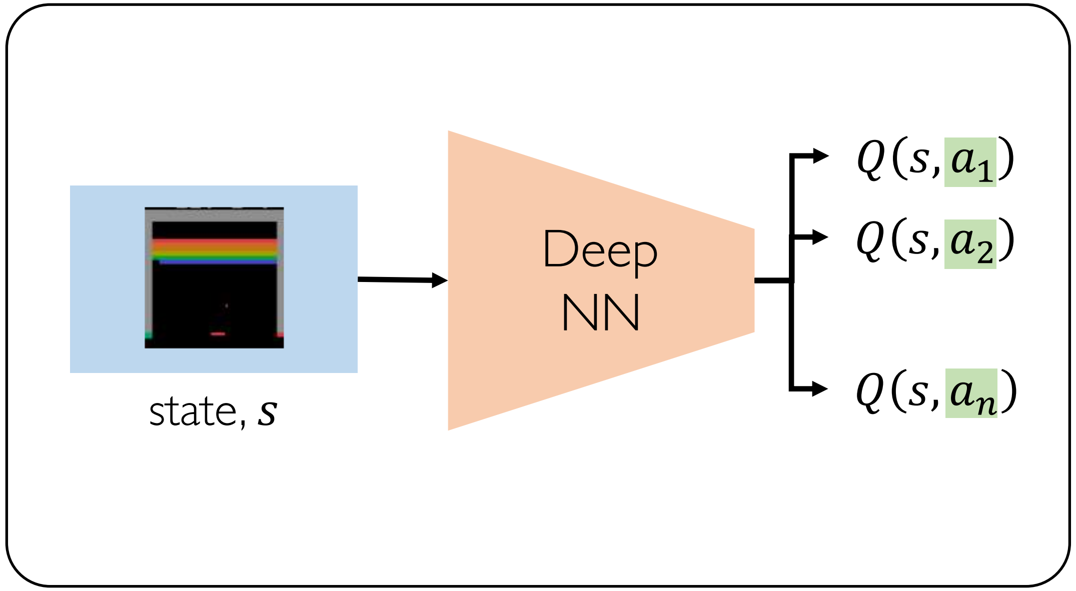

$$\mathcal{L} = \mathbb{E}\left[\left\|\left(r + \gamma \max_{a'} Q(s',a')\right) - Q(s,a)\right\|^2\right]$$

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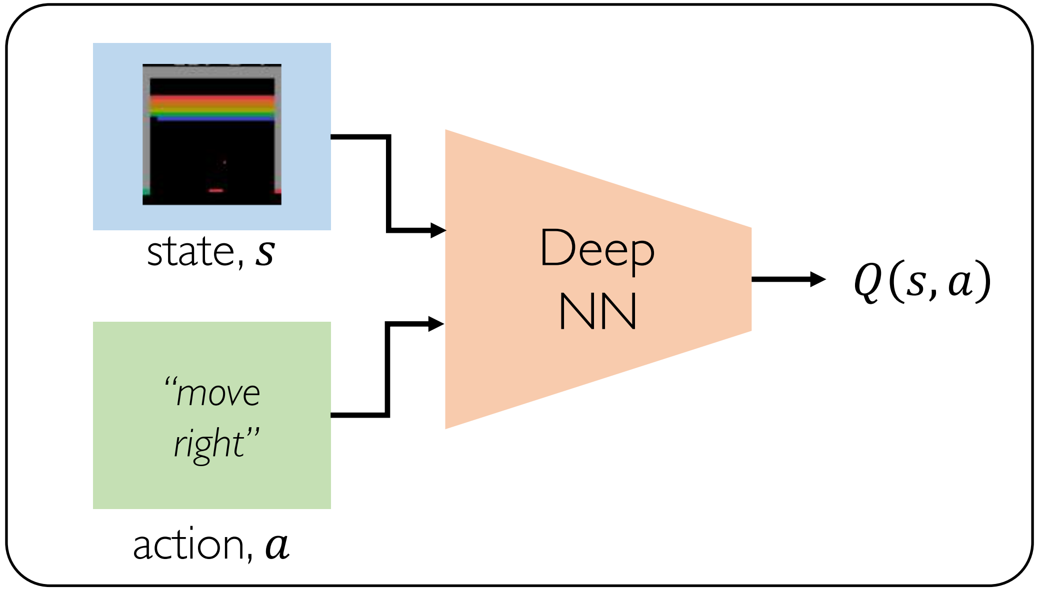

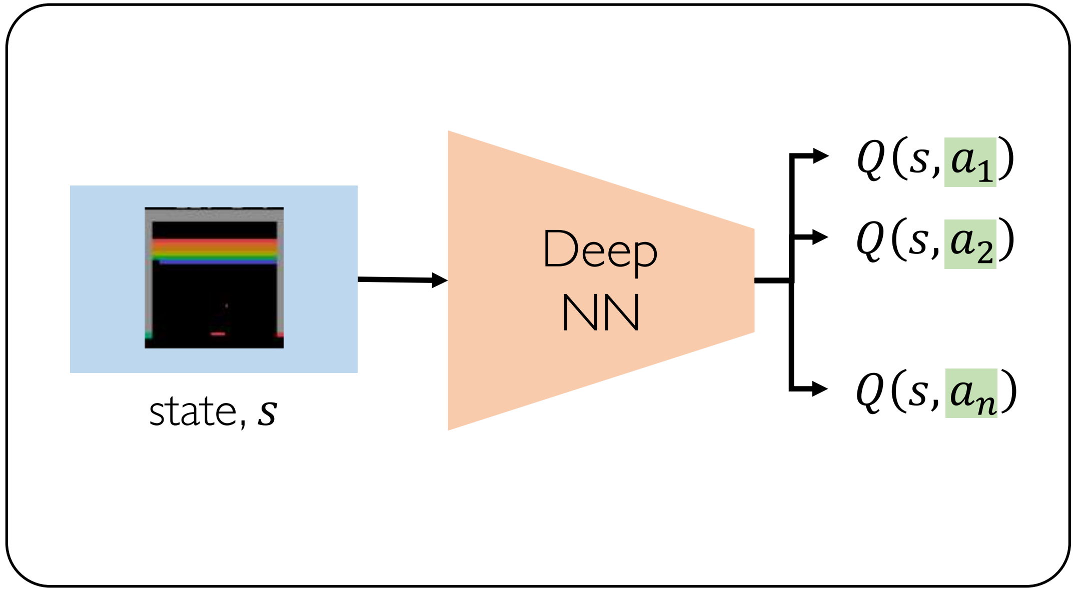

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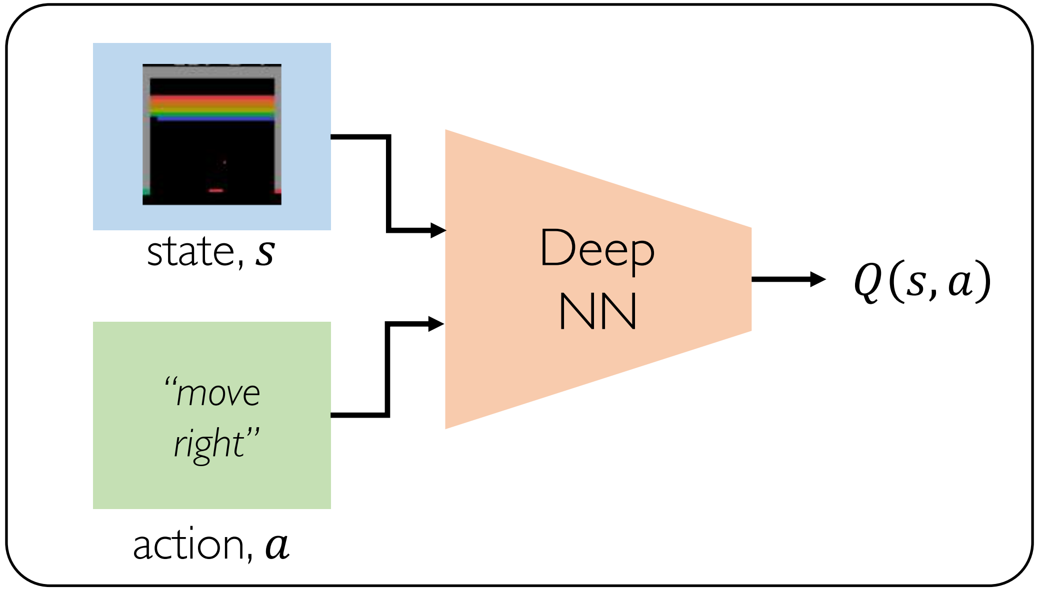

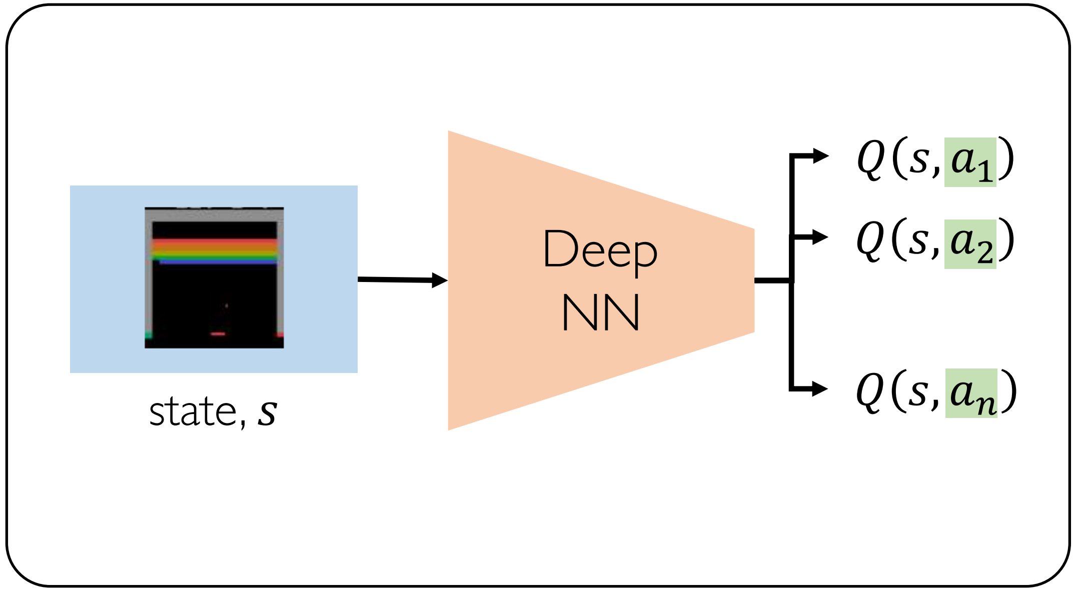

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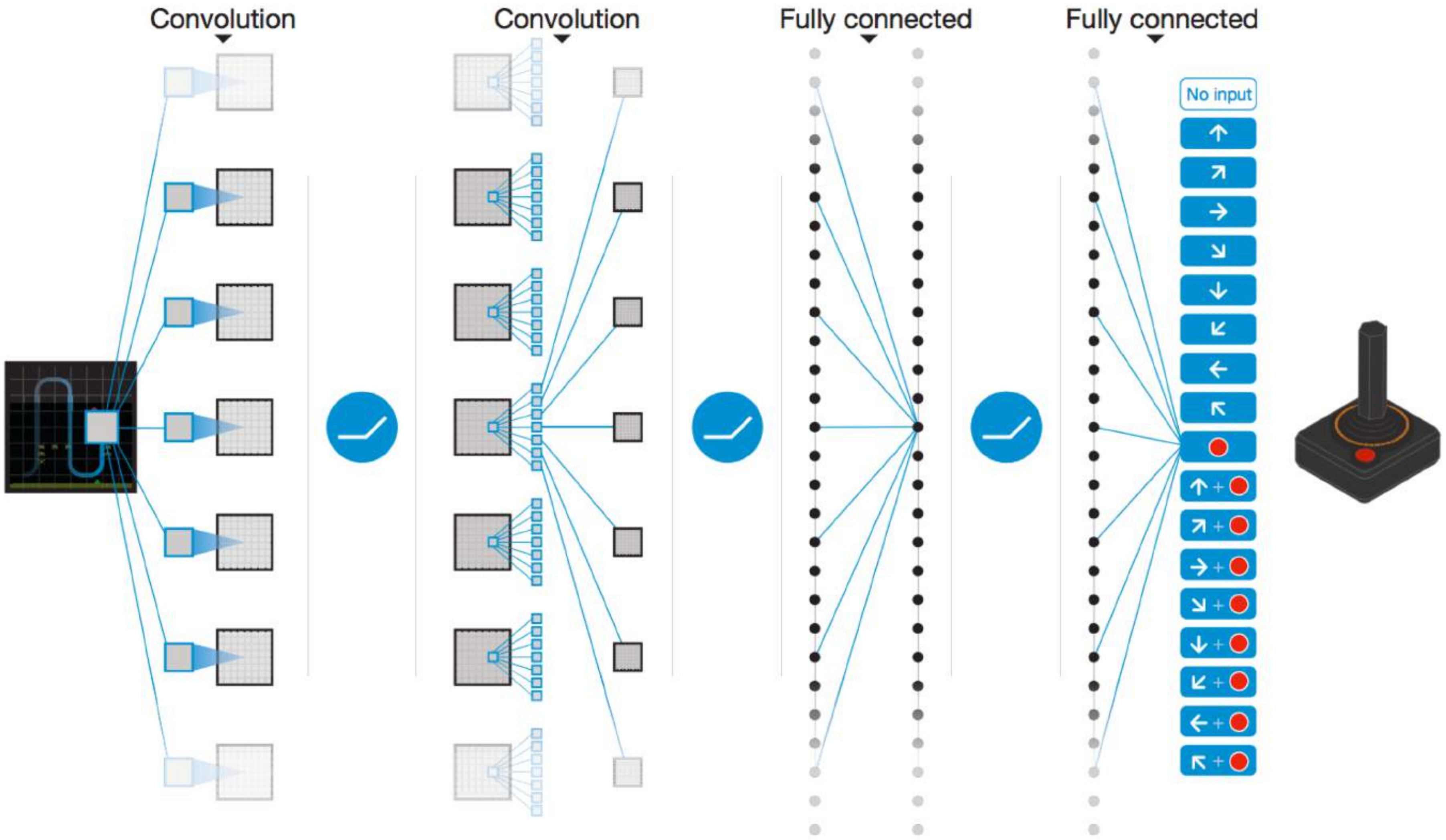
DQN Atari Results



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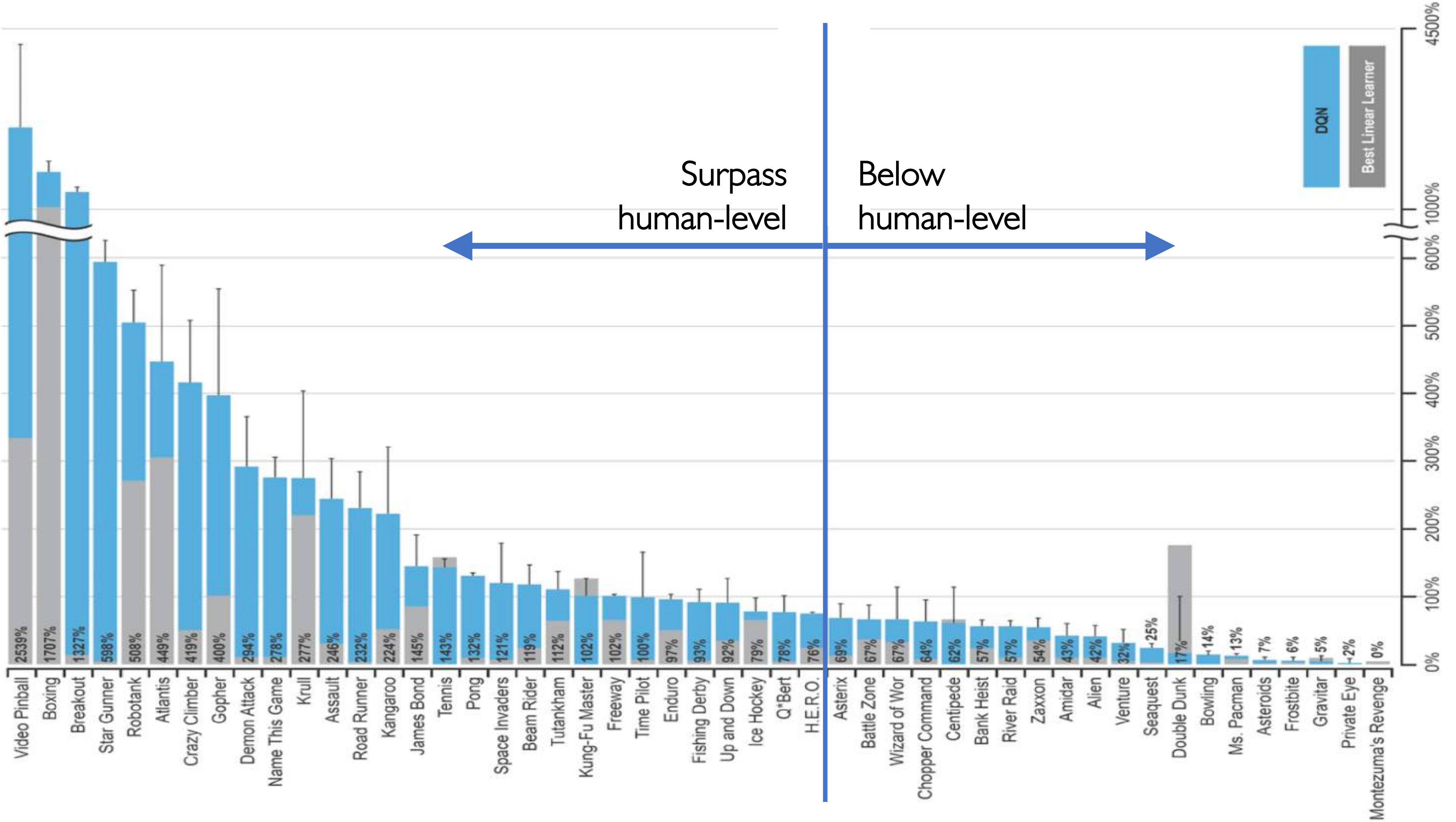


Downsides of Q-learning

Complexity:

- Can model scenarios where the action space is discrete and small
- Cannot handle continuous action spaces

IMPORTANT:
Imagine you want to predict steering wheel angle of a car!

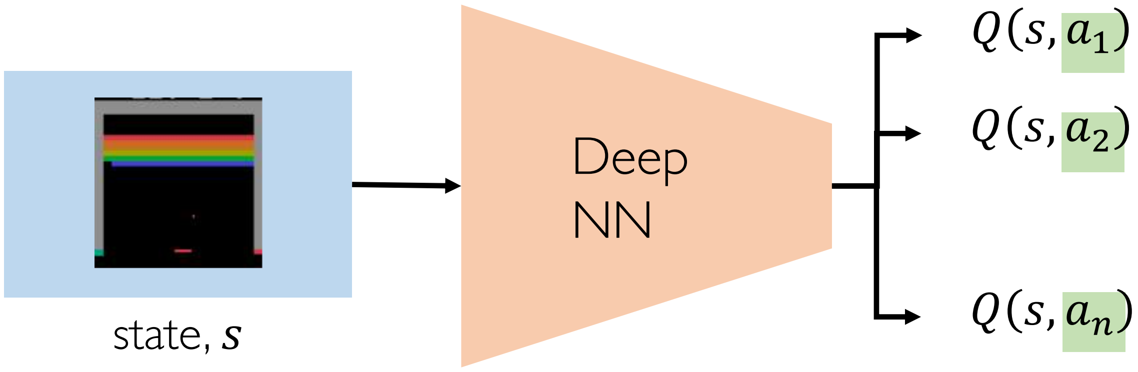
Flexibility:

- Cannot learn stochastic policies since policy is deterministically computed from the Q function

To overcome, consider a new class of RL training algorithms:
Policy gradient methods

Policy Gradient (PG) : Key Idea

DQN (before): Approximating Q and inferring the optimal policy,



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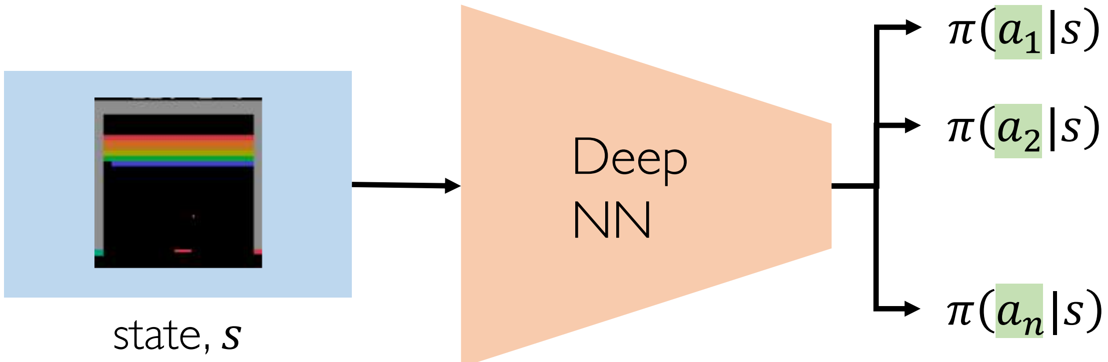

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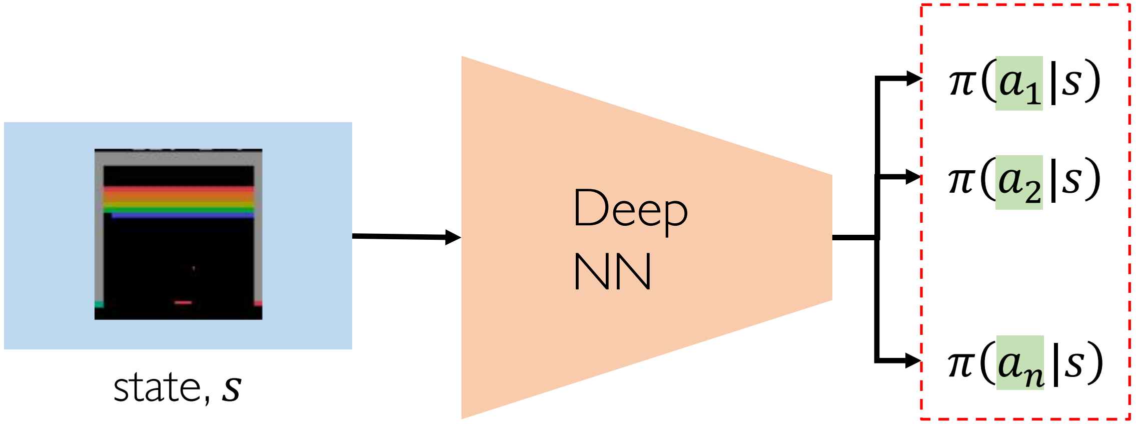

$$\sum_{a_i \in A} \pi(a_i|s) = 1$$

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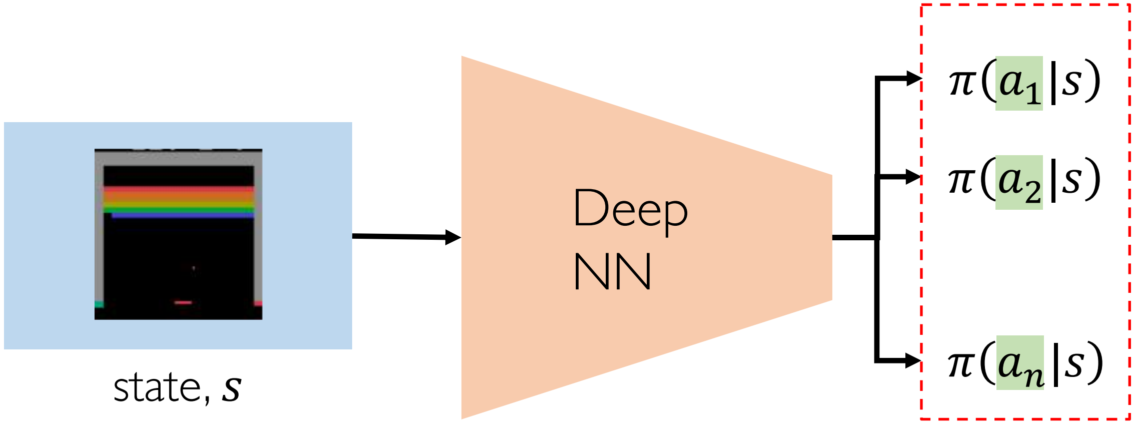

$$\sum_{a_i \in A} \pi(a_i|s) = 1$$

$$\pi(a|s) = P(action|state)$$

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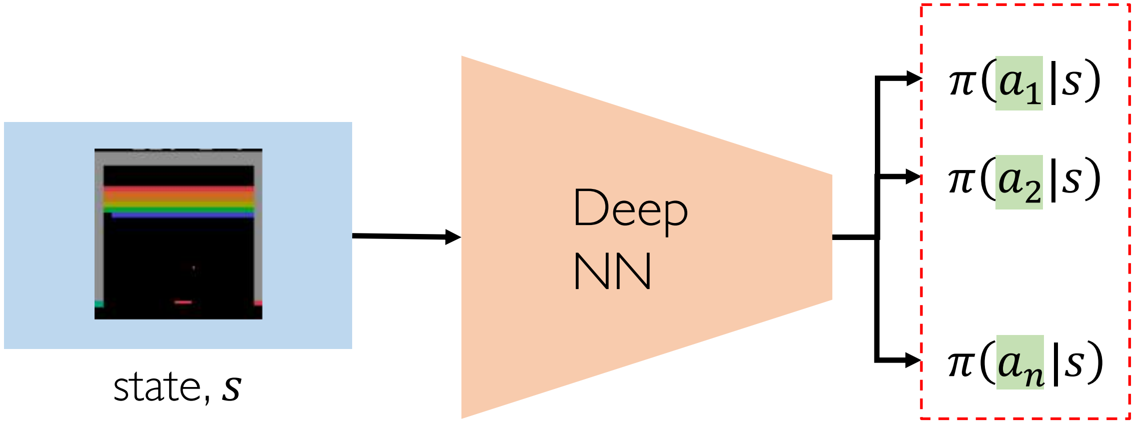

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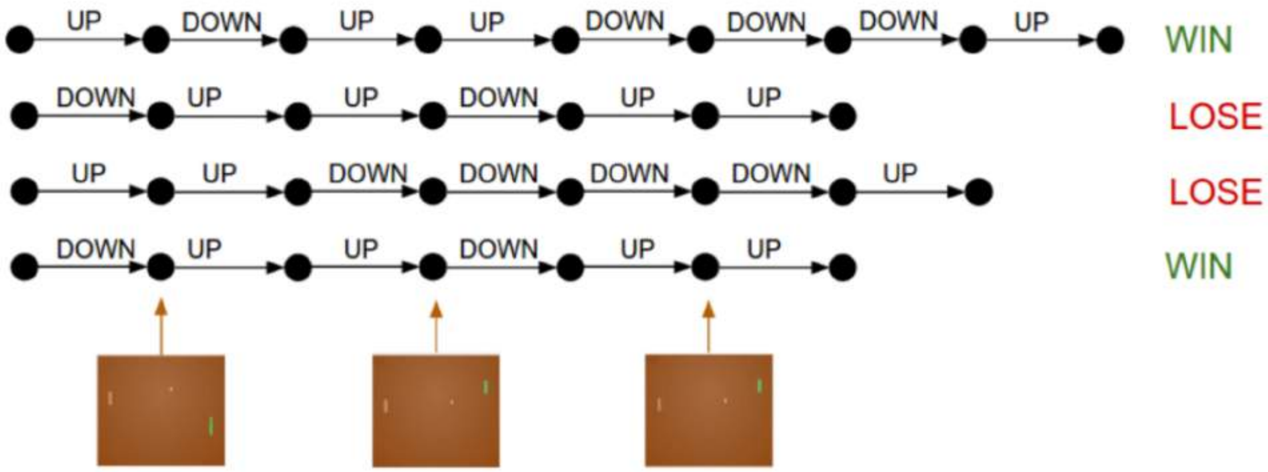

1. Run a policy for a while
2. Increase probability of actions that lead to high rewards
3. Decrease probability of actions that lead to low/no rewards

function REINFORCE
  Initialize θ
  for $episode \sim \pi_\theta$
    $\{s_i, a_i, r_i\}_{i=1}^{T-1} \leftarrow episode$
    for t = 1 to T-1
      $\nabla \leftarrow \nabla_\theta \log \pi_\theta(a_t|s_t)\, R_t$
      $\theta \leftarrow \theta + \alpha\nabla$
  return θ

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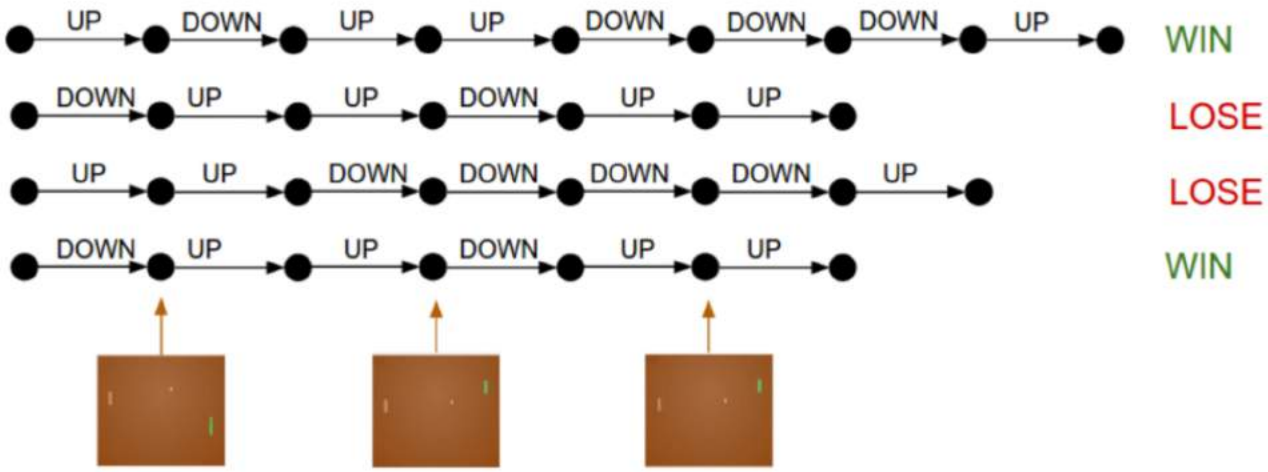

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    {s_i, a_i, r_i}_{i=1}^{T-1} ← episode
    for t = 1 to T-1
      ∇ ← ∇_θ log π_θ(a_t|s_t) R_t
      θ ← θ + α∇
  return θ
```

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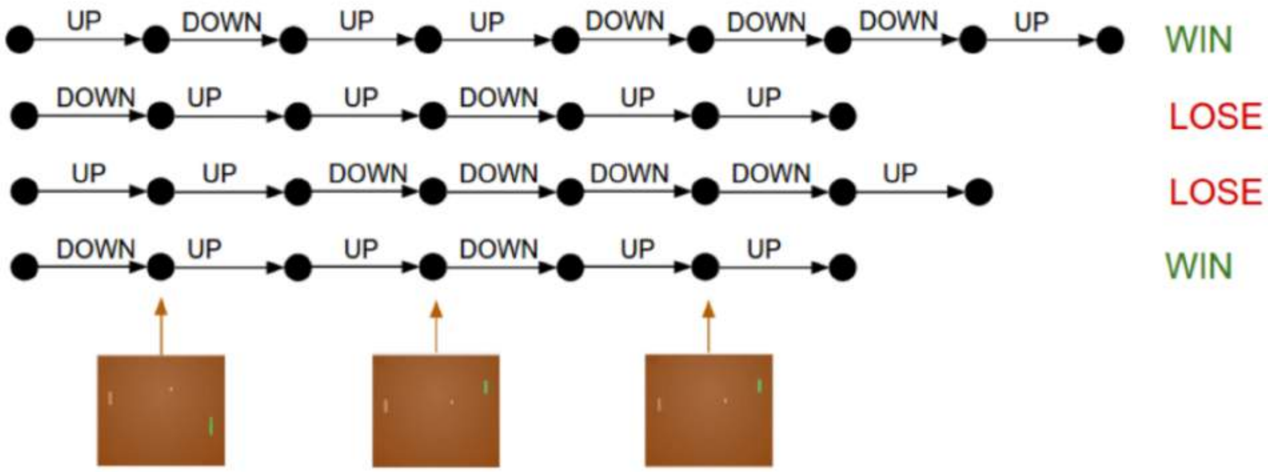

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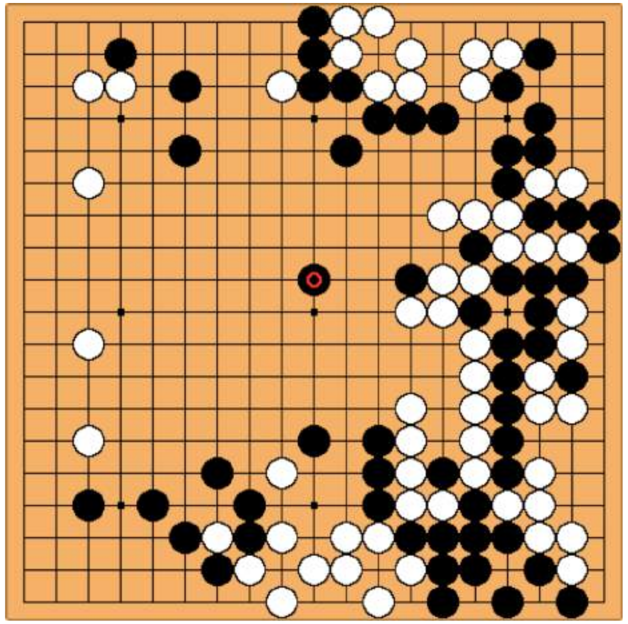
$$\nabla_\theta \underbrace{\log \pi_\theta(a_t|s_t)}_{\text{log-likelihood of action}} \underbrace{R_t}_{\text{reward}}$$

The Game of Go

Aim: Get more board territory than your opponent.



Board Size n x n	Positions 3^{n^2}	% Legal	Legal Positions
1×1	3	33.33%	1
2×2	81	70.37%	57
3×3	19,683	64.40%	12,675
4×4	43,046,721	56.49%	24,318,165
5×5	847,288,609,443	48.90%	414,295,148,741
9×9	$4.434264882 \times 10^{38}$	23.44%	$1.03919148791 \times 10^{38}$
13×13	$4.300233593 \times 10^{80}$	8.66%	$3.72497923077 \times 10^{79}$
19×19	$1.740896506 \times 10^{172}$	1.20%	$2.08168199382 \times 10^{170}$

Greater number of legal board positions than atoms in the universe.

Source: Wikipedia.

AlphaGo Beats Top Human Player at Go (2016)

Silver et al., *Nature* 2016.

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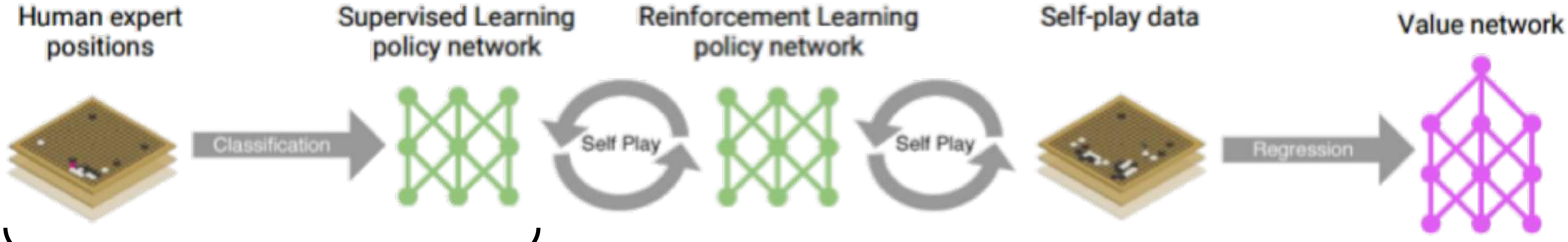

1) Initial training: human data

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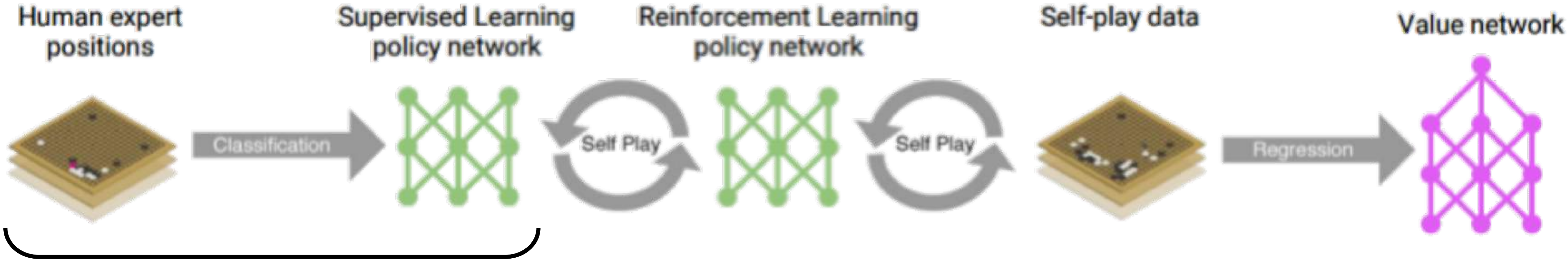

1) Initial training: human data

2) Self-play and reinforcement learning
→ super-human performance

Silver et al., *Nature* 2016.

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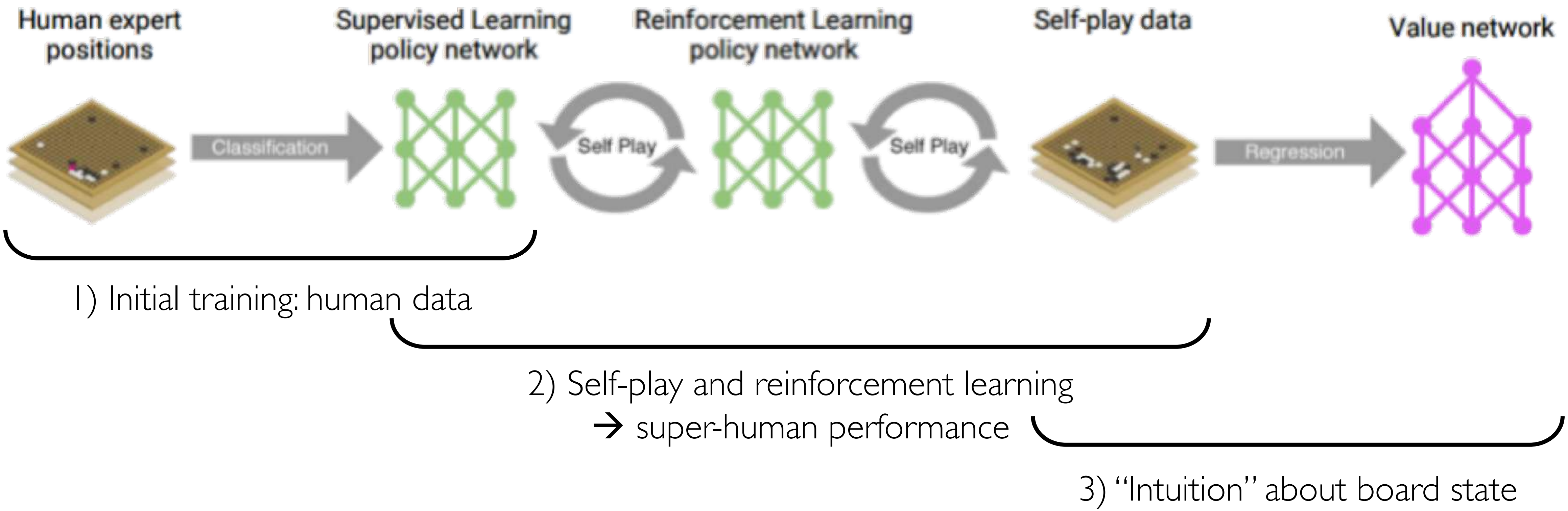

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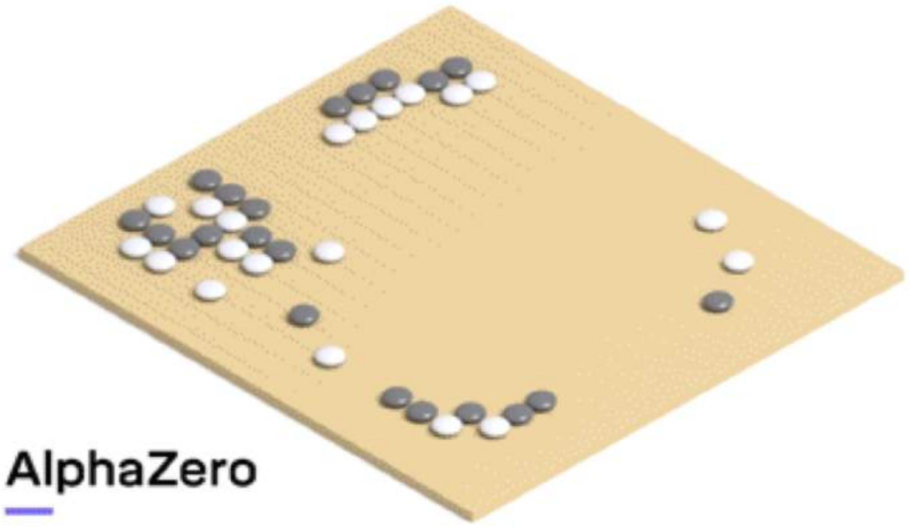
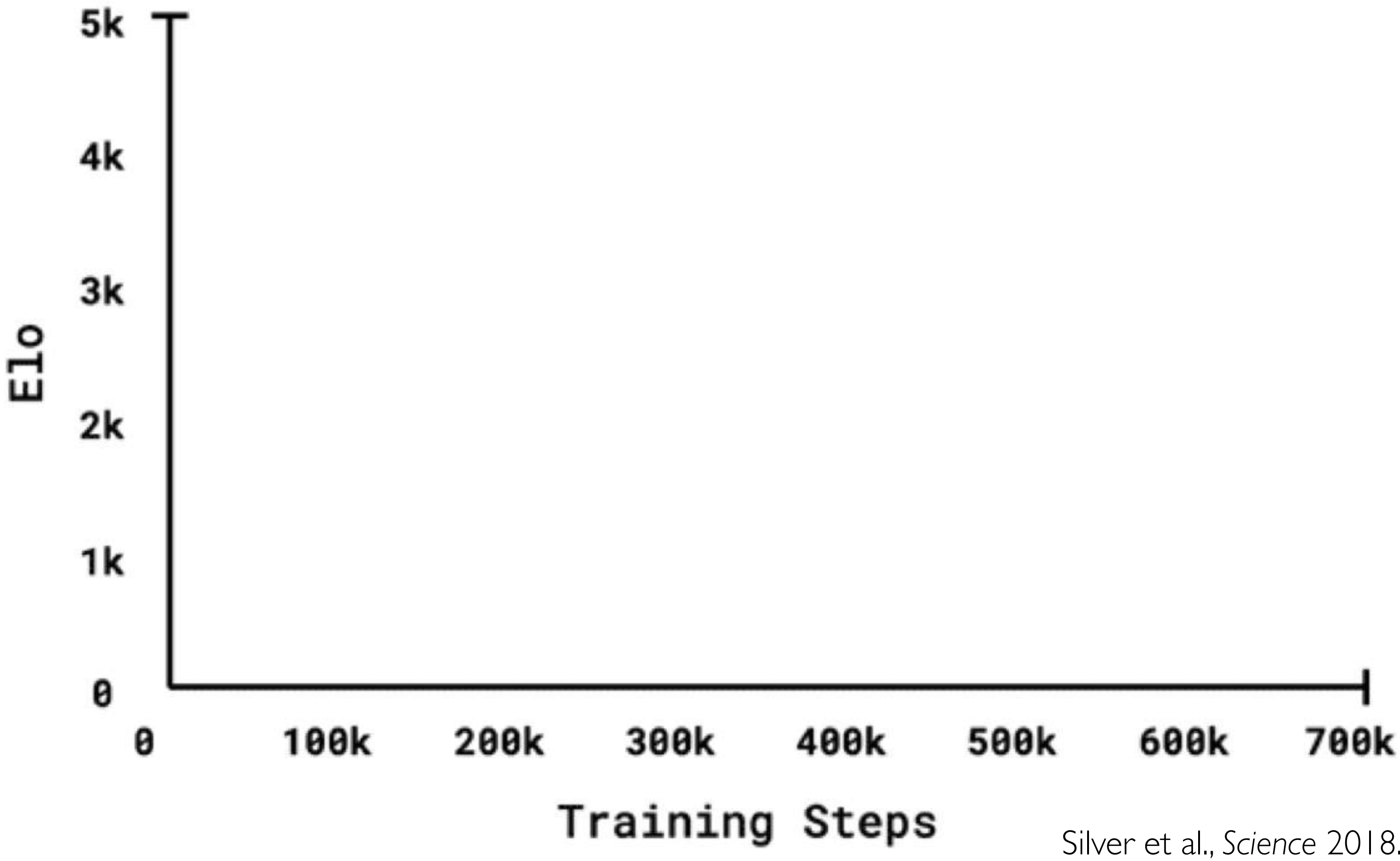
AlphaZero: RL from Self-Play (2018)





Silver et al., *Science* 2018.

Questions?