Deep Reinforcement Learning

Alexander Amini

MIT Introduction to Deep Learning

January 10, 2024

Learning in Dynamic Environments

Reinforcement Learning: Robots, Games, the World

Robotics

Game Play and Strategy

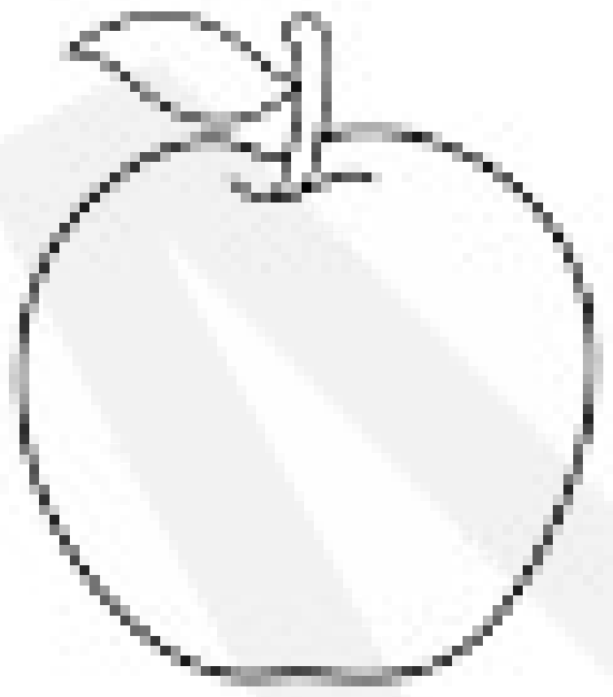
Classes of Learning Problems

Supervised Learning

Data: (x, y)
x is data, y is label

Goal: Learn function to map
$$x \rightarrow y$$

Apple example:



This thing is an apple.

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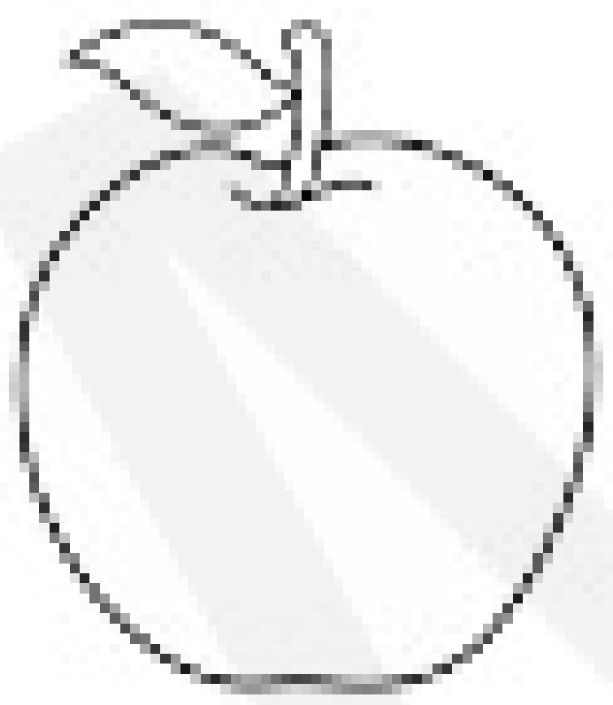

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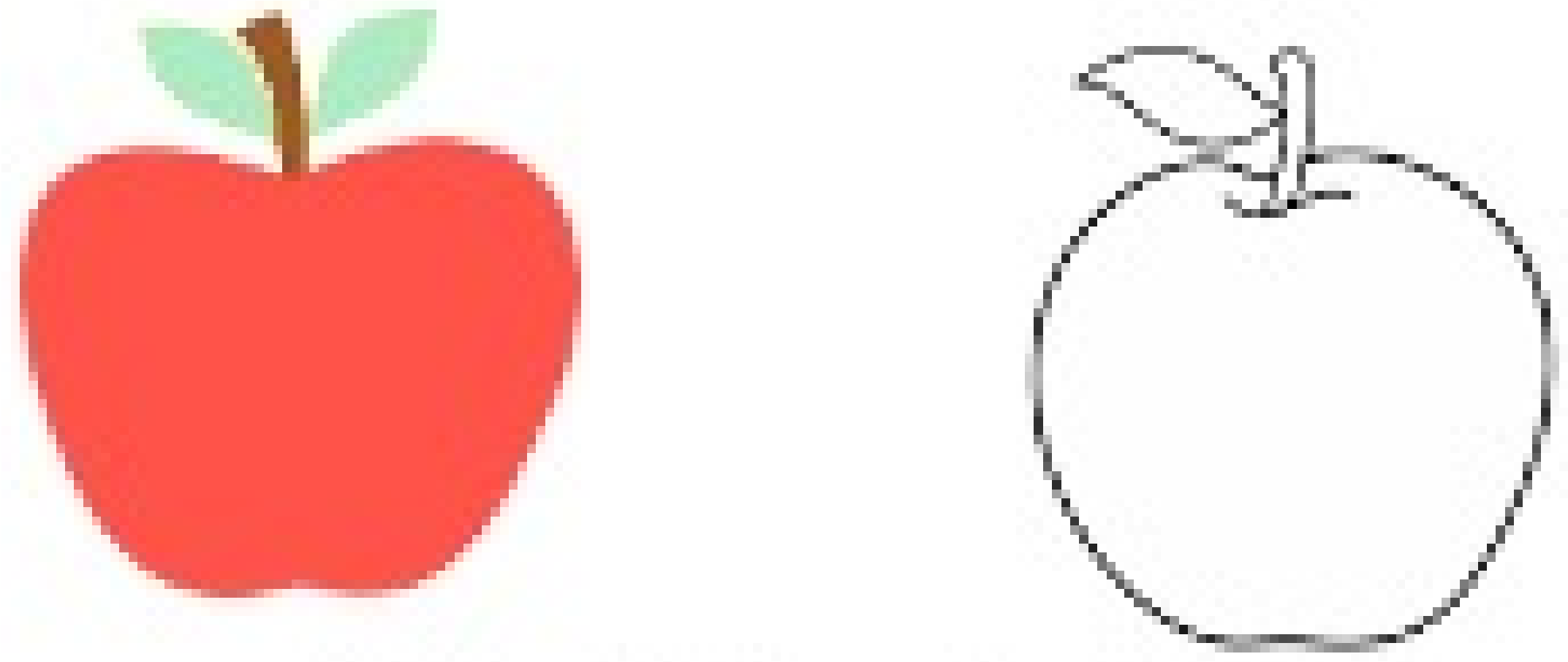
Unsupervised Learning

Data: x
x is data, no labels!

Goal: Learn underlying structure

Apple example:



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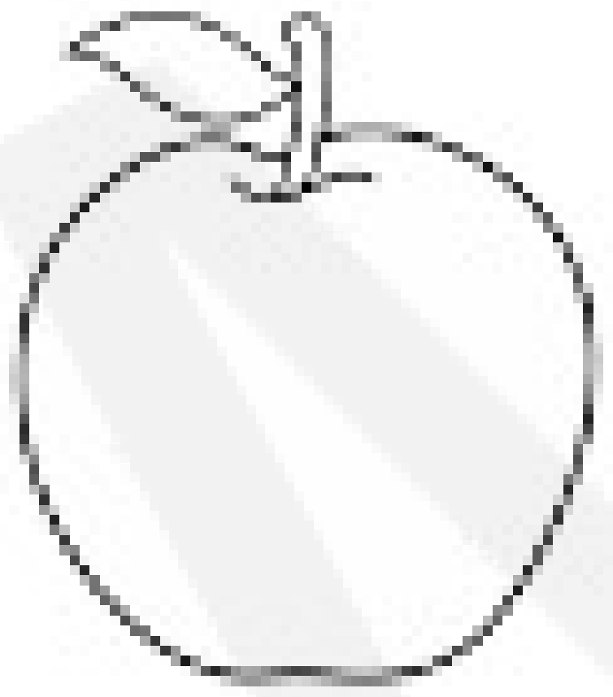

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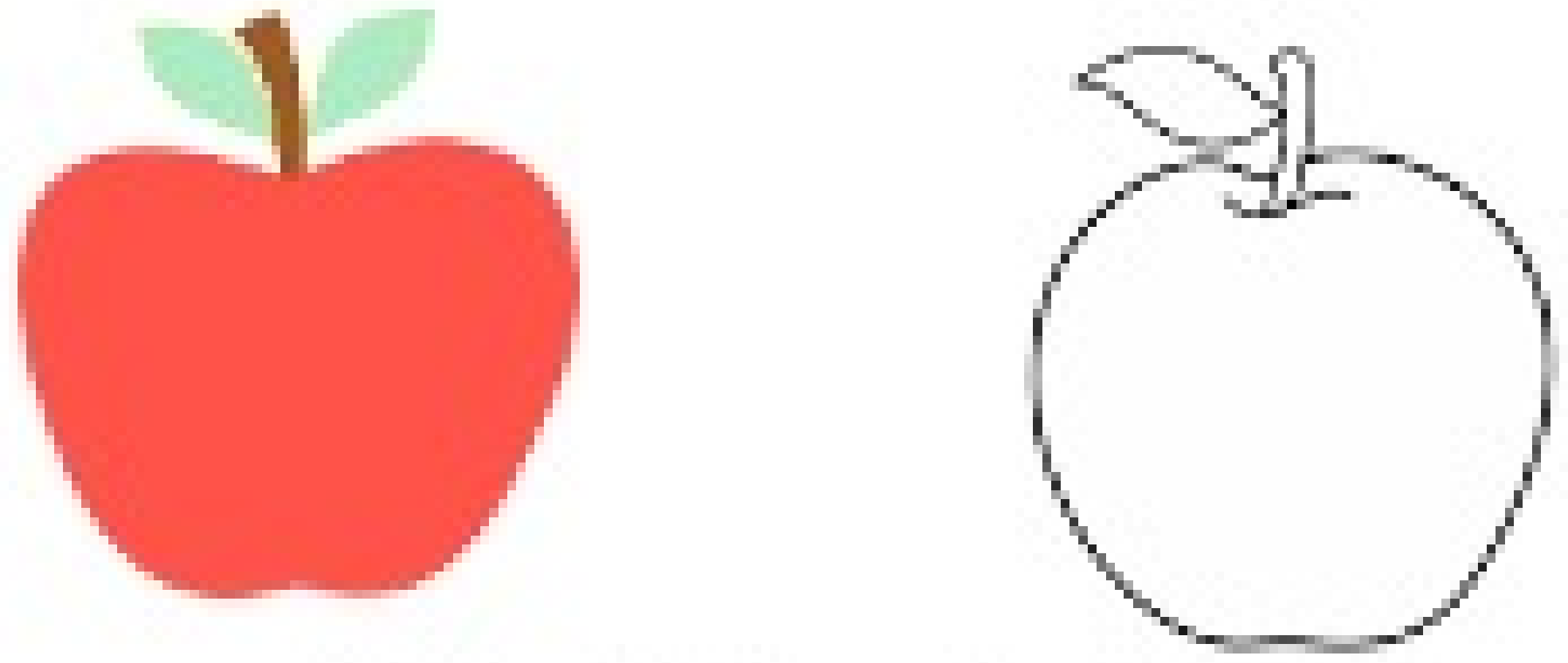

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Reinforcement Learning

Data: state-action pairs

Goal: Maximize future rewards over many time steps

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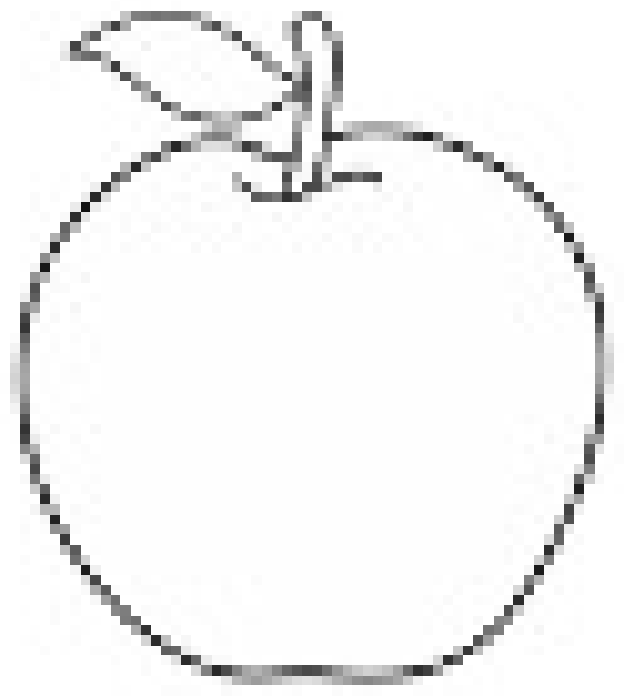

Eat this thing because it
will keep you alive.

Classes of Learning Problems

Supervised Learning

Data: (x, y)
x is data, y is label

Goal: Learn function to map
$x \to y$

Apple example:

This is an apple.

Unsupervised Learning

Data: x
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This thing is like the other thing.

Reinforcement Learning

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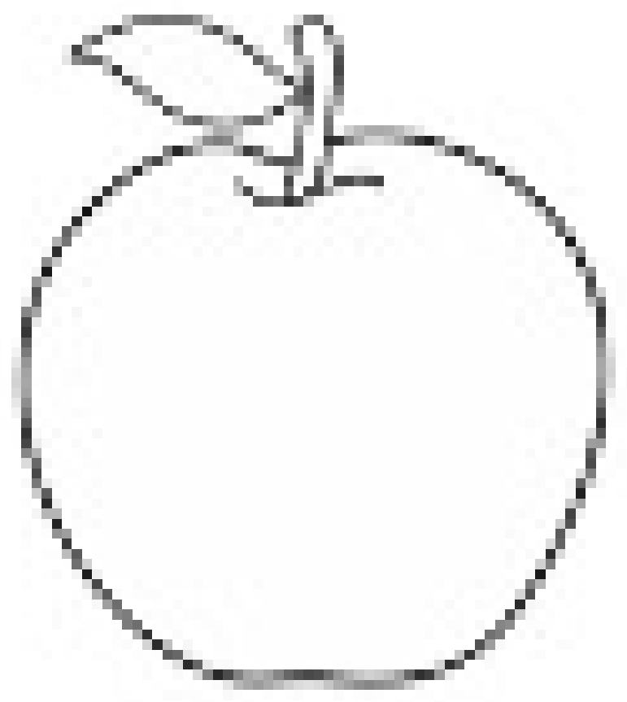

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RL: our focus today.

Reinforcement Learning (RL): Key Concepts

Agent: takes actions.

Reinforcement Learning (RL): Key Concepts

Environment: the world in which the agent exists and operates.

Reinforcement Learning (RL): Key Concepts

Action: a move the agent can make in the environment.

Action space A: the set of possible actions an agent can make in the environment

Reinforcement Learning (RL): Key Concepts

Observations: of the environment after taking actions.

Reinforcement Learning (RL): Key Concepts

State: a situation which the agent perceives.

Reinforcement Learning (RL): Key Concepts

Reward: feedback that measures the success or failure of the agent's action.

Reinforcement Learning (RL): Key Concepts

OBSERVATIONS

State changes: s_{t+1}

Reward: r_t

AGENT

ENVIRONMENT

Action: a_t

ACTIONS

Total Reward (Return)

$$R_t = \sum_{i=t}^{\infty} r_i$$

Reinforcement Learning (RL): Key Concepts

OBSERVATIONS

State changes: s_{t+1}

Reward: r_t

AGENT

ENVIRONMENT

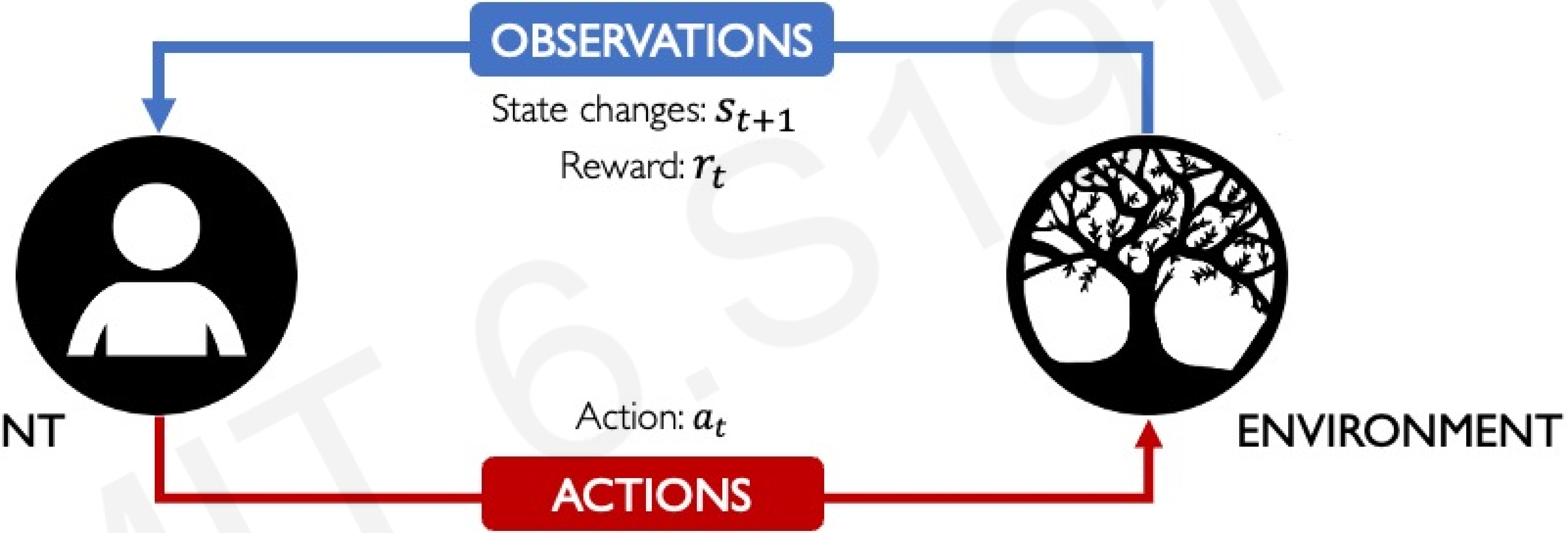
Action: a_t

ACTIONS

Total Reward (Return)

$$R_t = \sum_{i=t}^{\infty} r_i = r_t + r_{t+1} \ldots + r_{t+n} + \cdots$$

Reinforcement Learning (RL): Key Concepts



Discounted Total Reward (Return)

$$R_t = \sum_{i=t}^{\infty} \gamma^i r_i$$

Reinforcement Learning (RL): Key Concepts

OBSERVATIONS

State changes: s_{t+1}

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ENVIRONMENT

Action: a_t

ACTIONS

Discounted Total Reward (Return)

$$R_t = \sum_{i=t}^{\infty} \gamma^i r_i = \gamma^t r_t + \gamma^{t+1} r_{t+1} \ldots + \gamma^{t+n} r_{t+n} + \cdots$$

γ: discount factor; $0 < \gamma < 1$

Defining the Q-function

$$R_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots$$

Total reward, R_t, is the discounted sum of all rewards obtained from time t

$$Q(s_t, a_t) = \mathbb{E}[R_t | s_t, a_t]$$

The Q-function captures the **expected total future reward** an agent in state, s, can receive by executing a certain action, a

How to take actions given a Q-function?

$$Q(s_t, a_t) = \mathbb{E}[R_t | s_t, a_t]$$

(state, action)

Ultimately, the agent needs a **policy** $\boldsymbol{\pi(s)}$, to infer the **best action to take** at its state, s

Strategy: the policy should choose an action that maximizes future reward

$$\pi^*(s) = \underset{a}{\operatorname{argmax}}\, Q(s, a)$$

Deep Reinforcement Learning Algorithms

Value Learning

Find $Q(s, a)$

$$a = \underset{a}{\operatorname{argmax}}\, Q(s, a)$$

Policy Learning

Find $\pi(s)$

Sample $a \sim \pi(s)$

Deep Reinforcement Learning Algorithms

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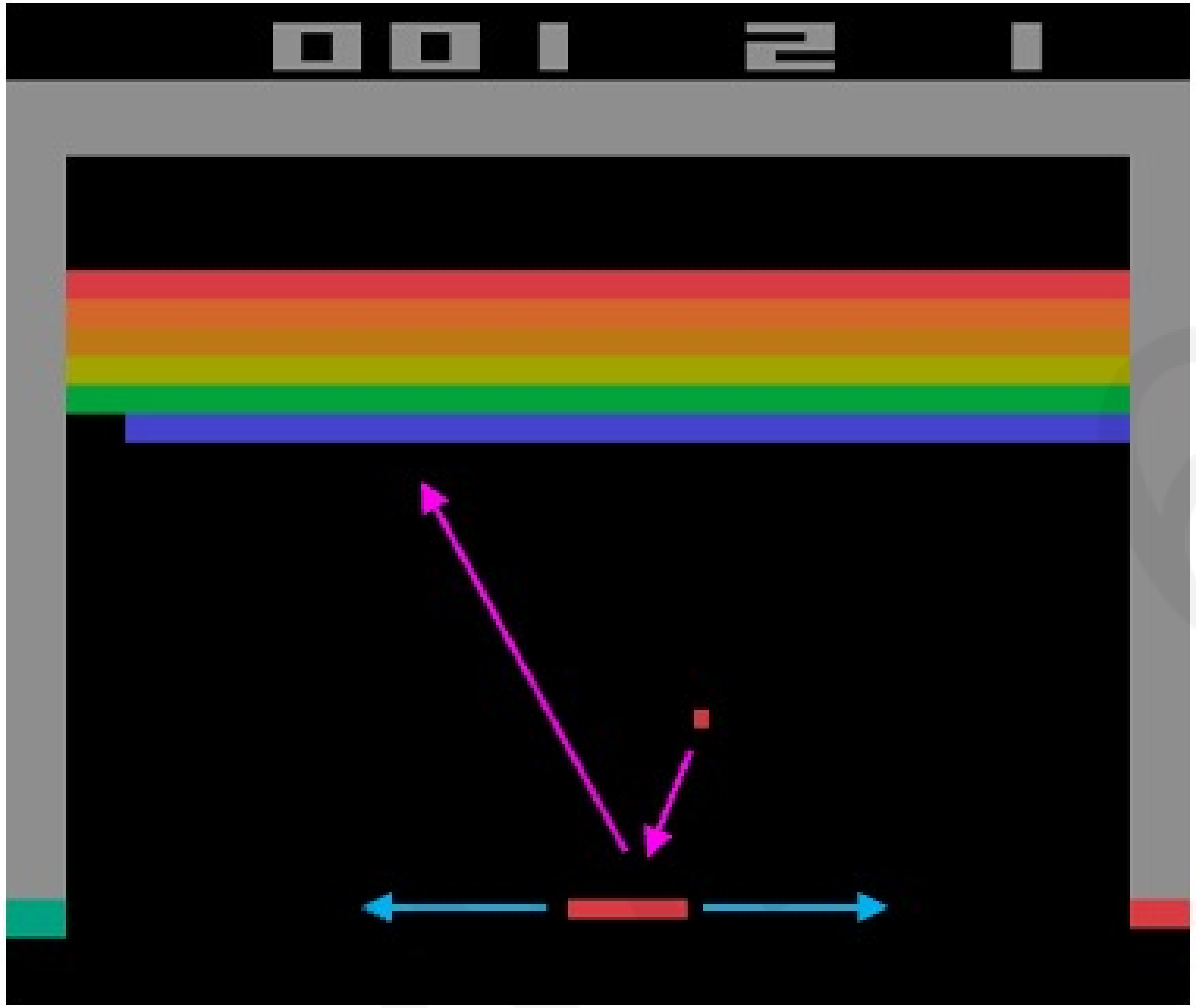
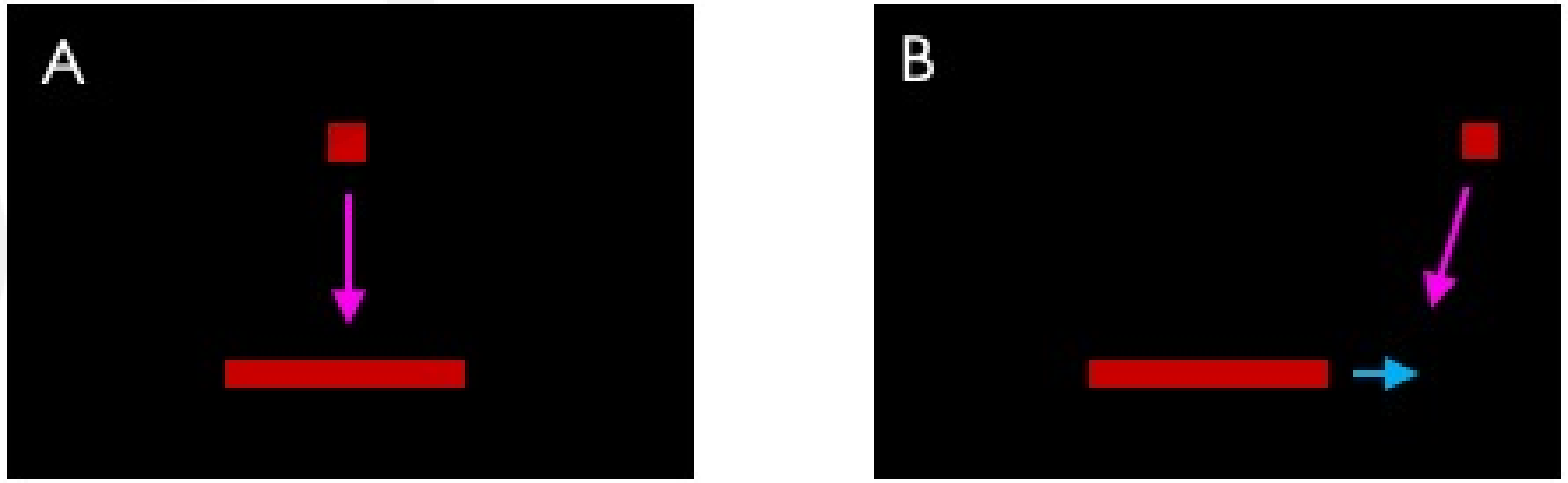
Digging deeper into the Q-function

Example: Atari Breakout



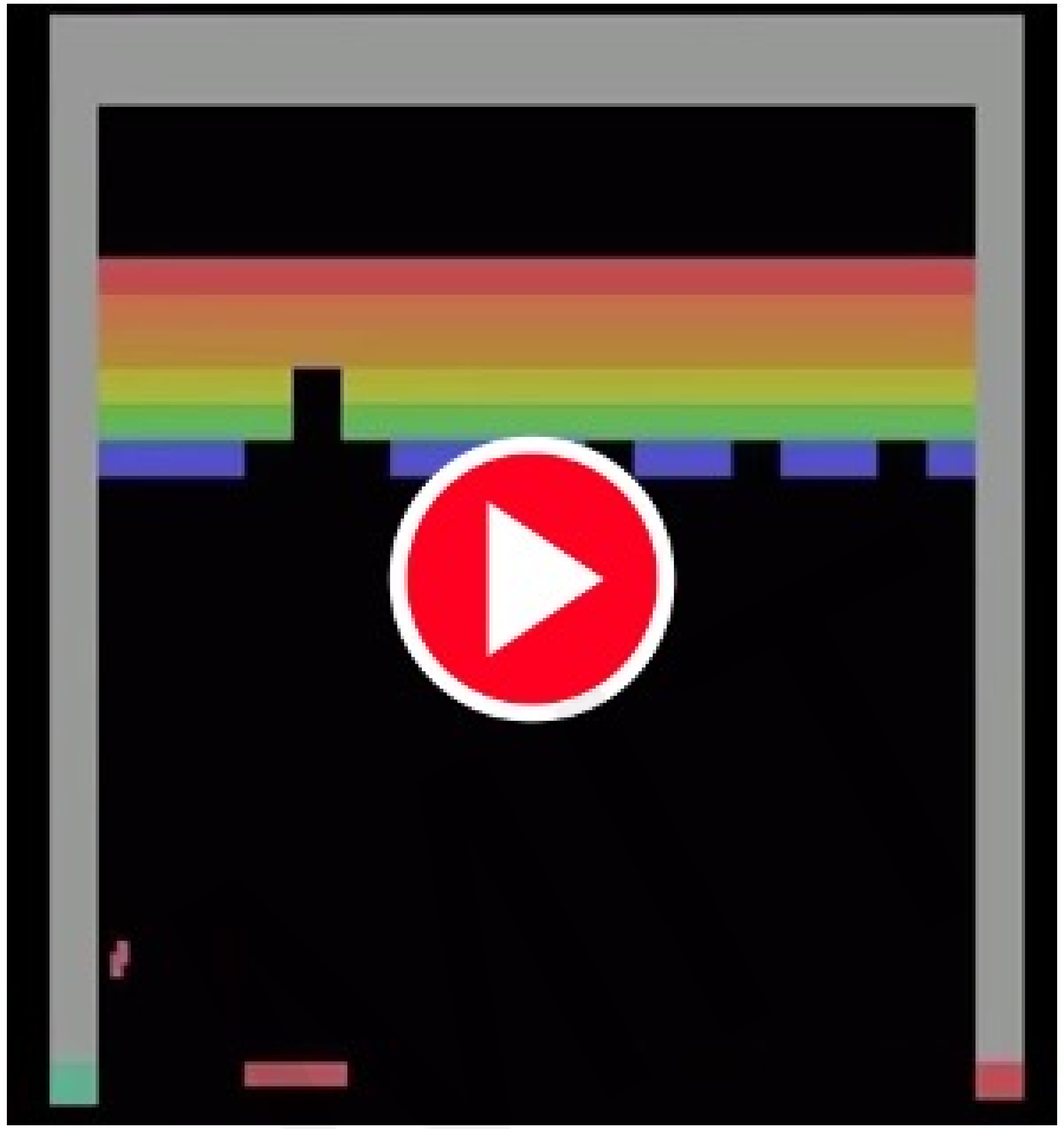
It can be very difficult for humans to accurately estimate Q-values



Which (s, a) pair has a higher Q-value? 🤔

Digging deeper into the Q-function

Example: Atari Breakout - Middle



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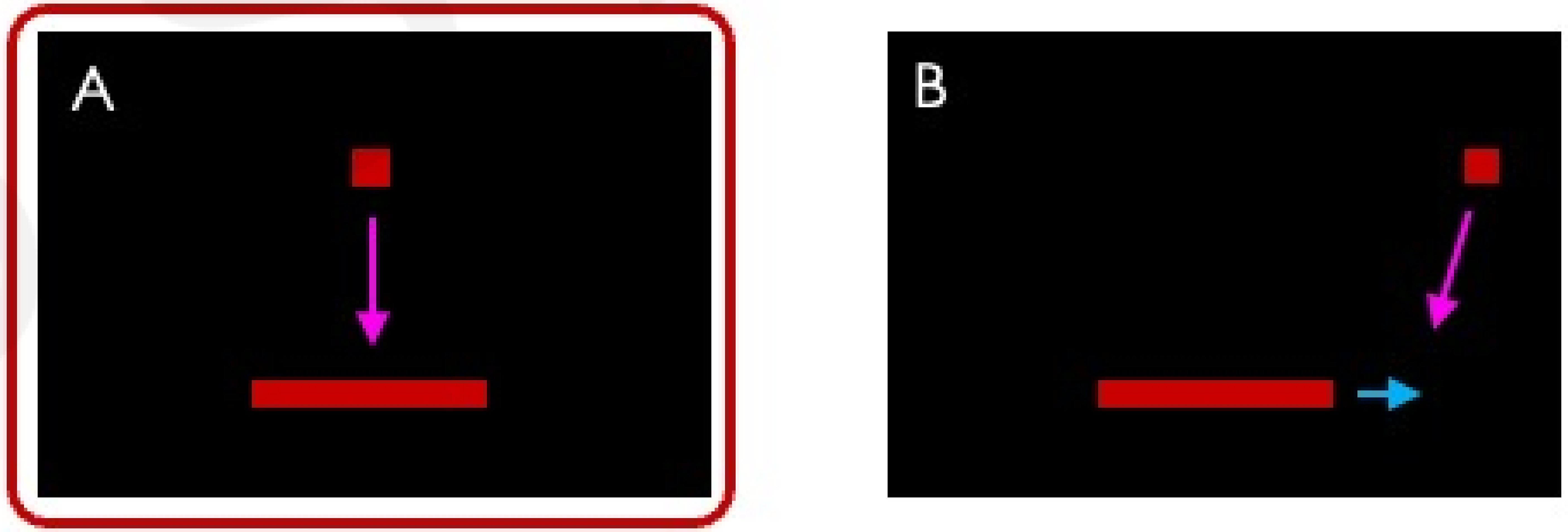
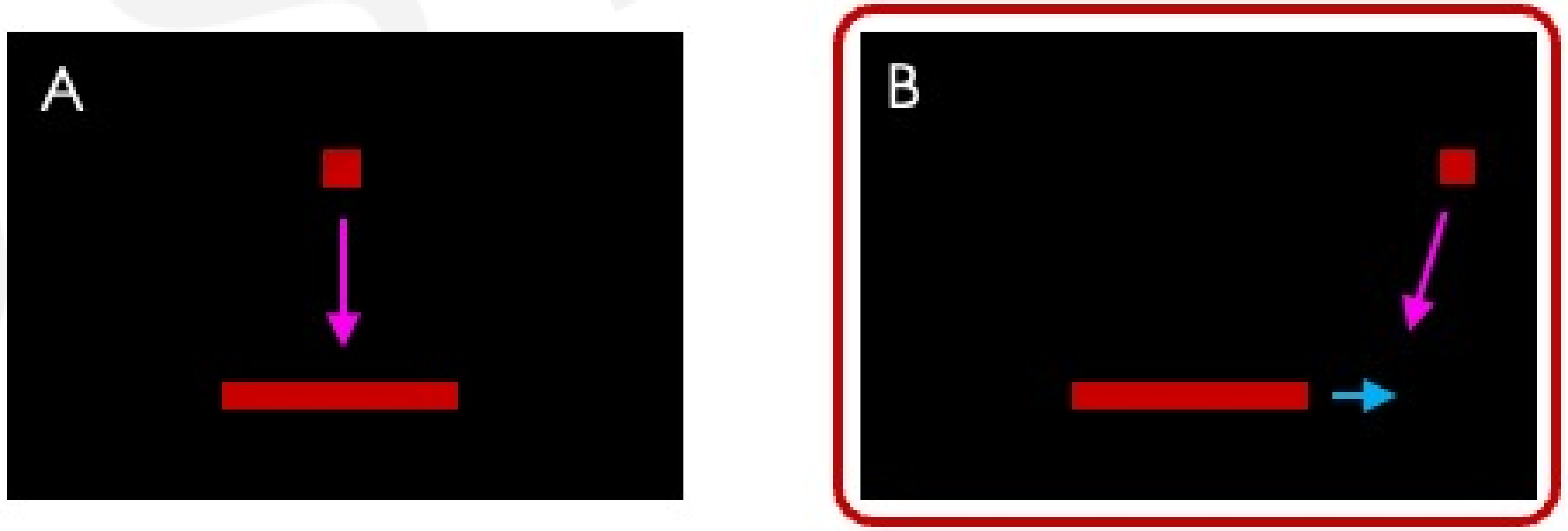


Which (**s**, **a**) pair has a higher Q-value? 🤔

Digging deeper into the Q-function

Example: Atari Breakout - Side

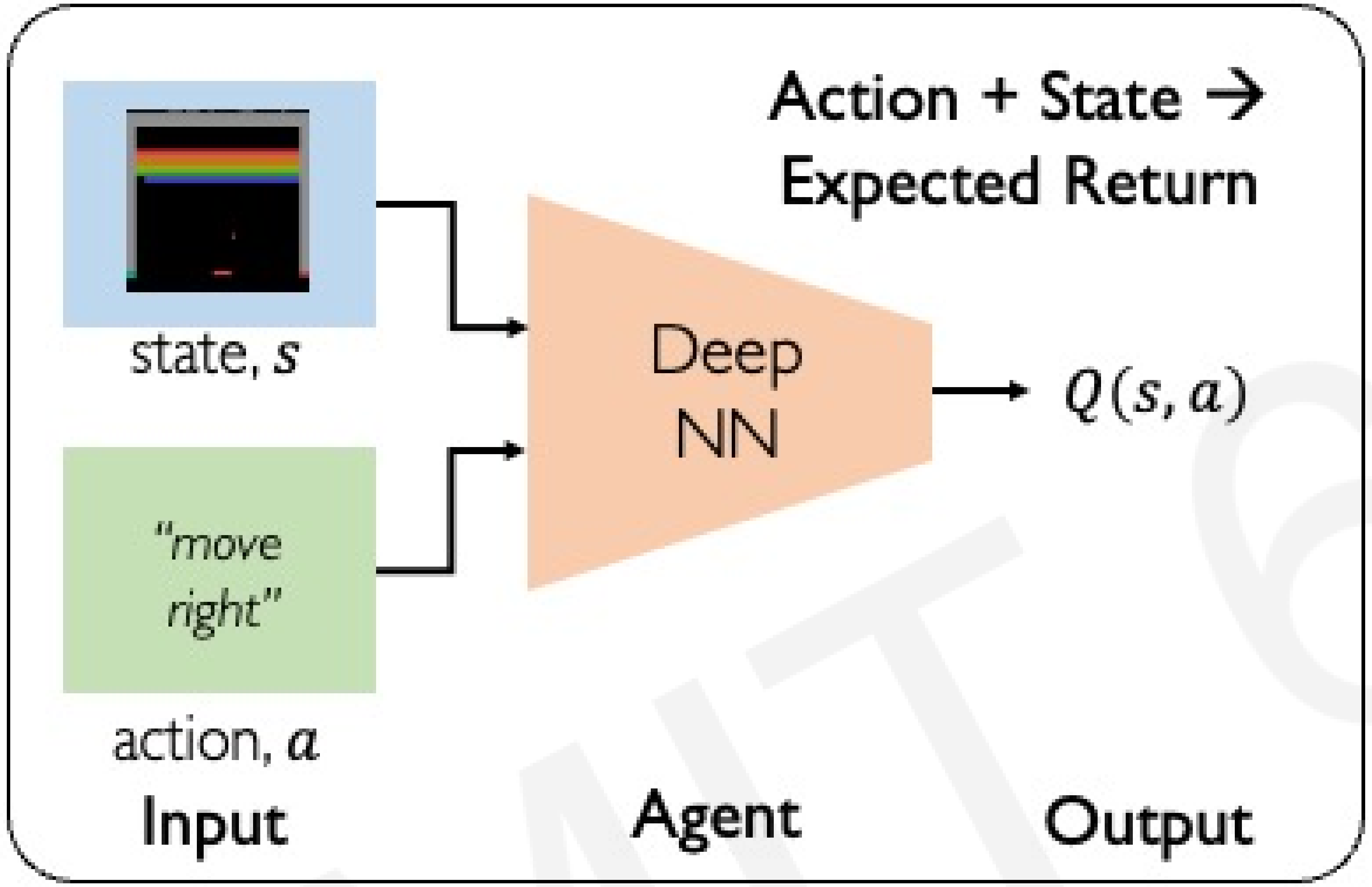
It can be very difficult for humans to accurately estimate Q-values



Which (s, a) pair has a higher Q-value? 🤔

Deep Q Networks (DQN)

How can we use deep neural networks to model Q-functions?



Mnih+ *Nature* 2015.

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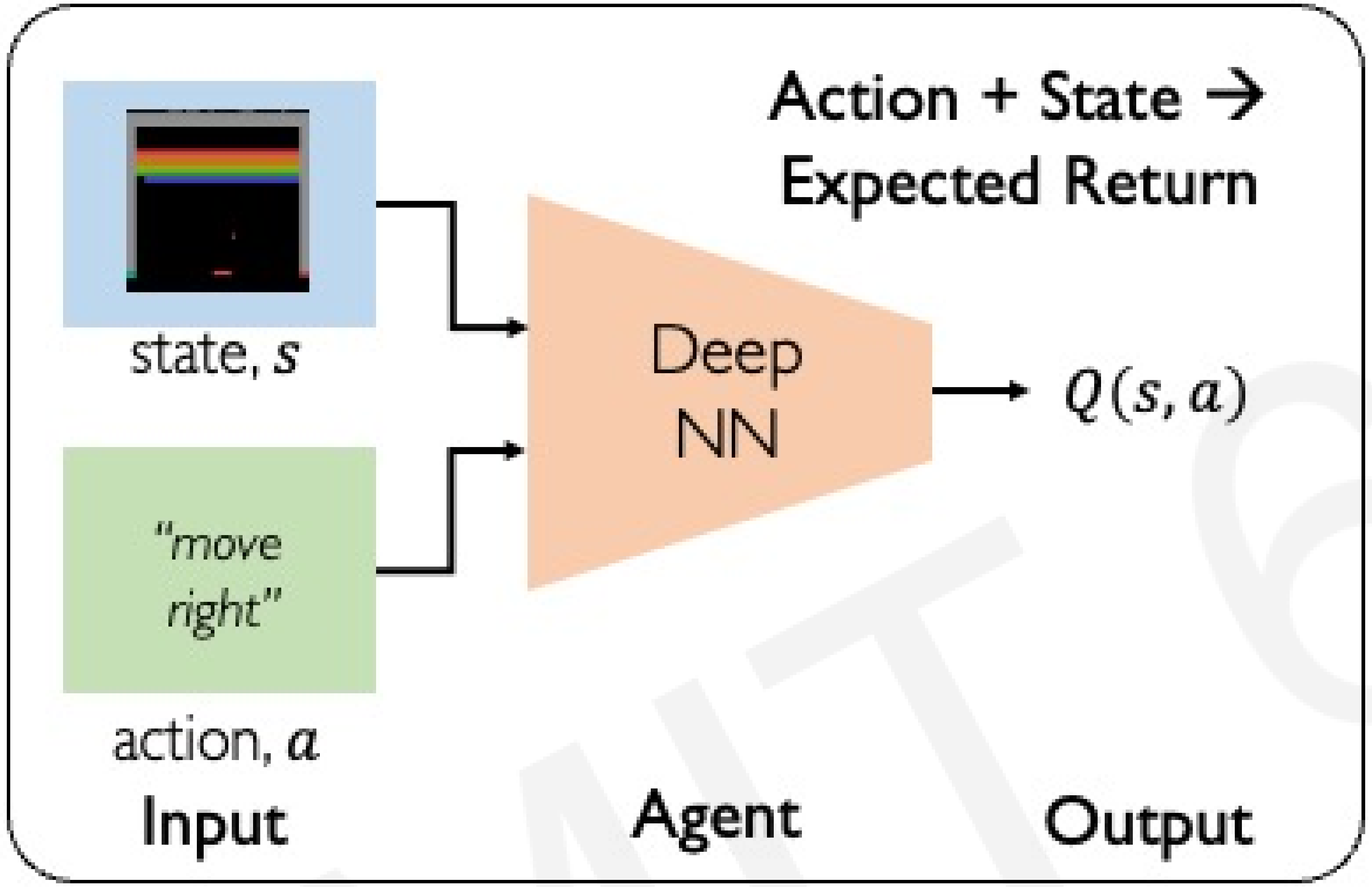

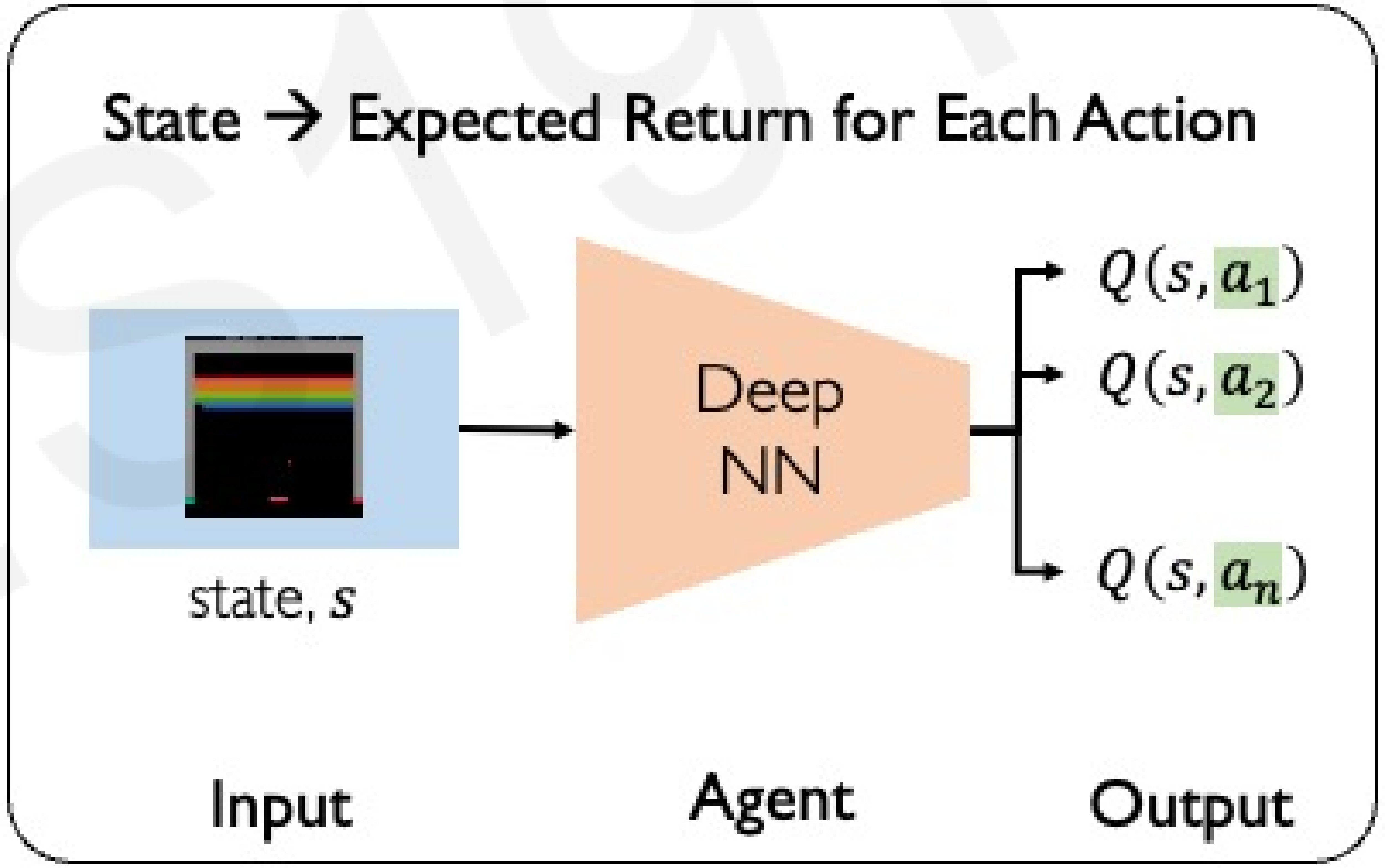

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Deep Q Networks (DQN): Training

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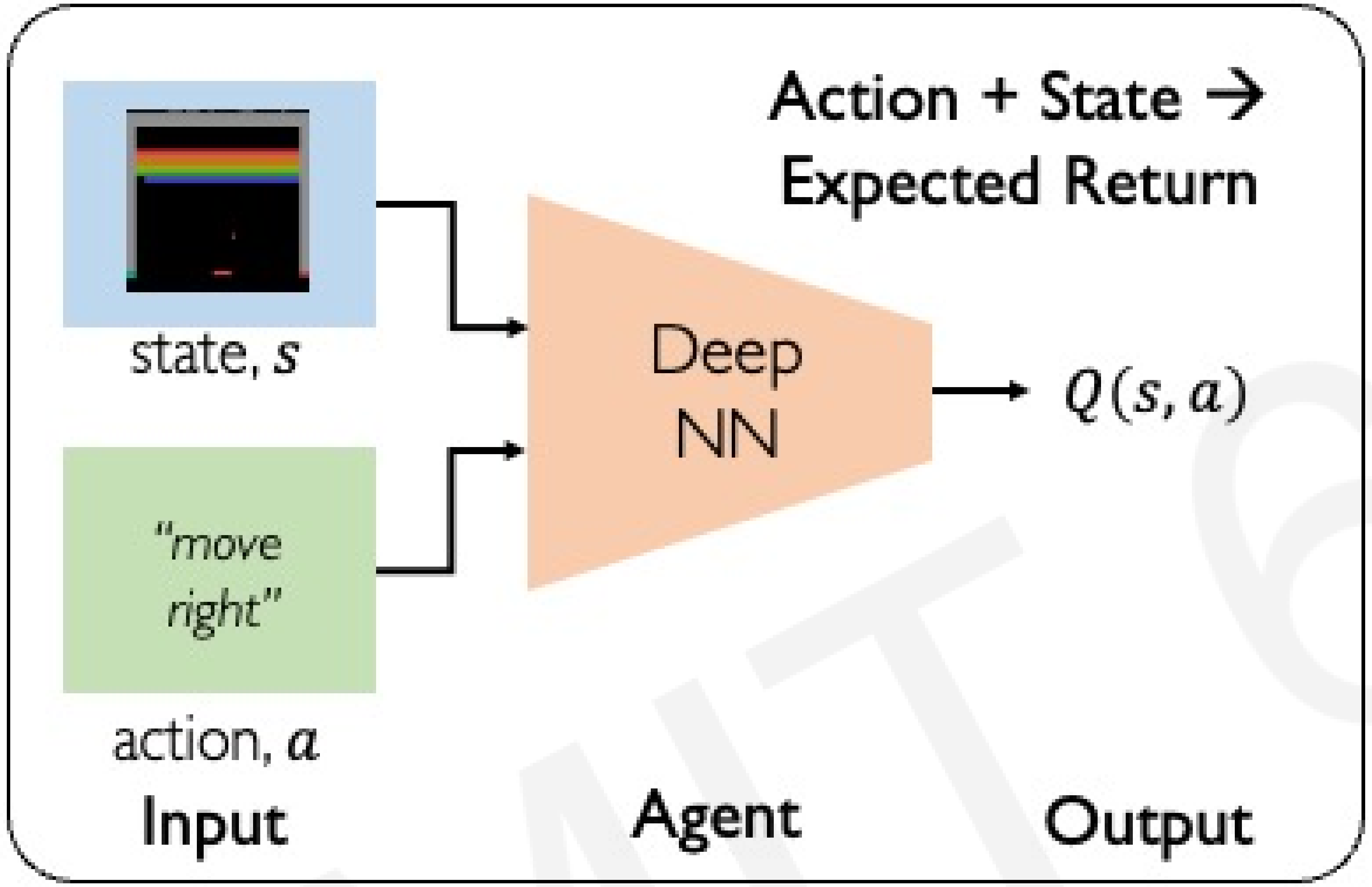


Action + State → Expected Return

state, s

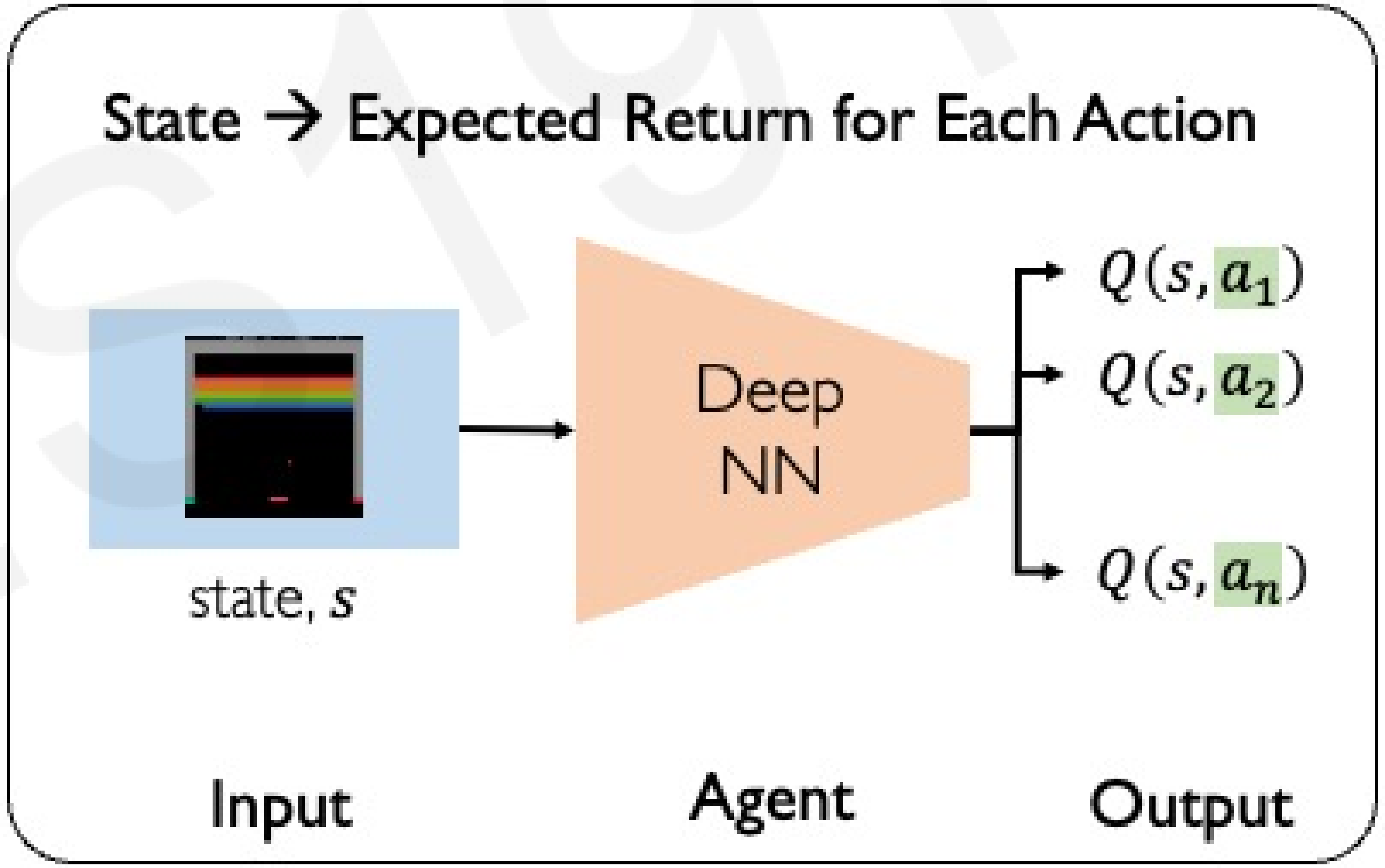
"move right"

action, a

Deep NN

$Q(s, a)$

Input **Agent** **Output**



State → Expected Return for Each Action

state, s

Deep NN

$Q(s, a_1)$

$Q(s, a_2)$

$Q(s, a_n)$

Input **Agent** **Output**

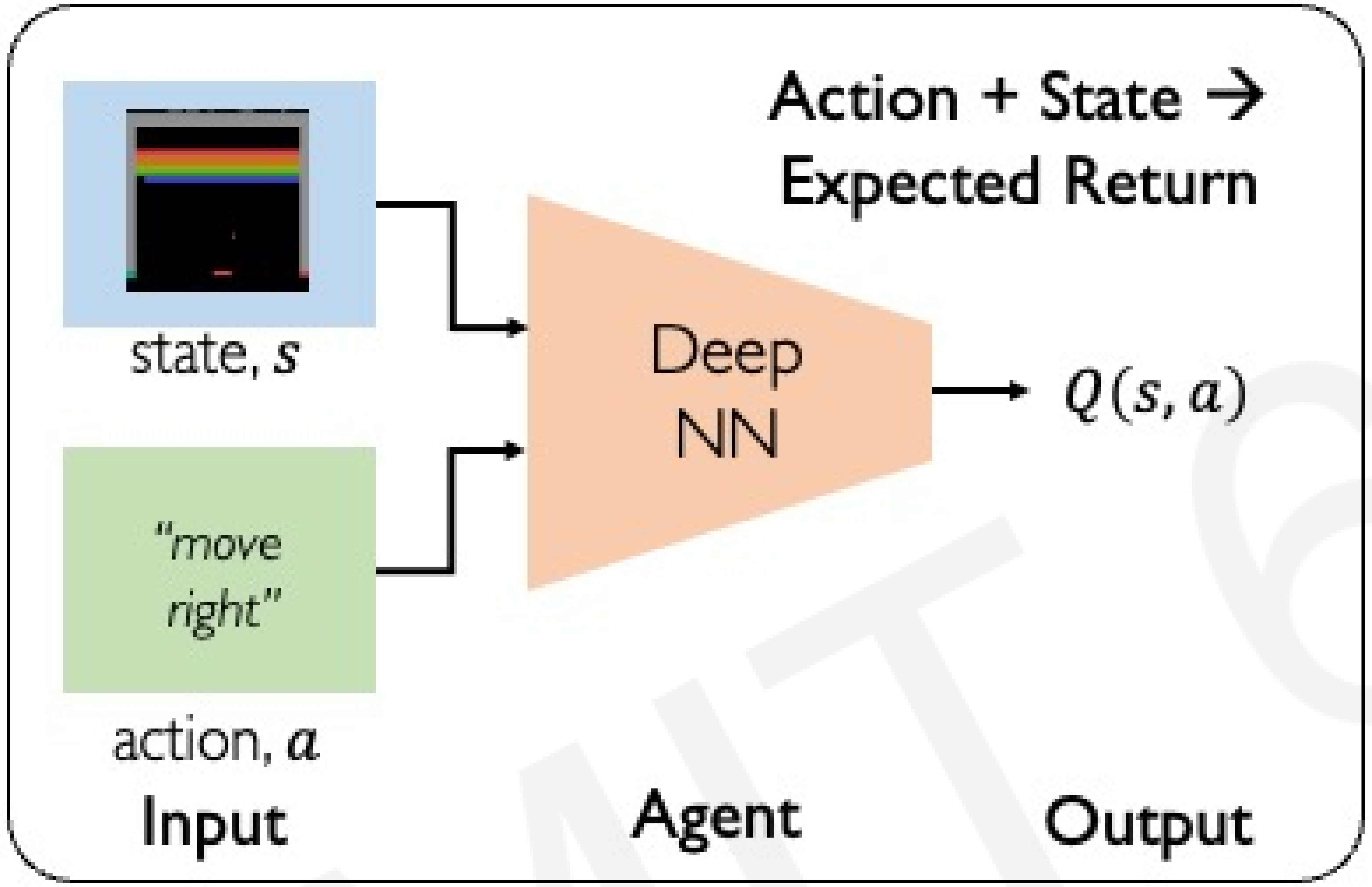
What happens if we take all the best actions?

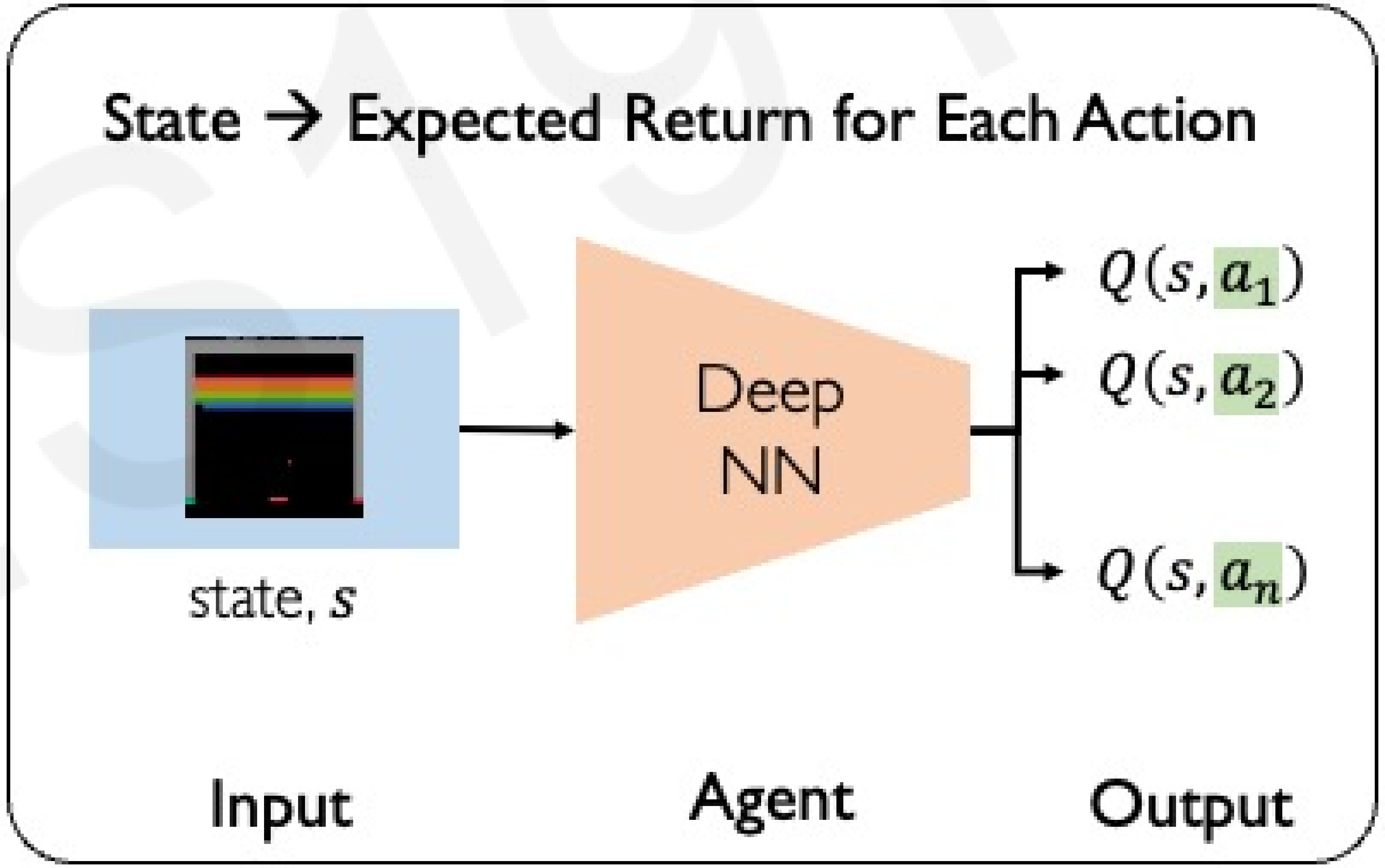
Maximize target return → train the agent

Mnih+ *Nature* 2015.

Deep Q Networks (DQN): Training

How can we use deep neural networks to model Q-functions?





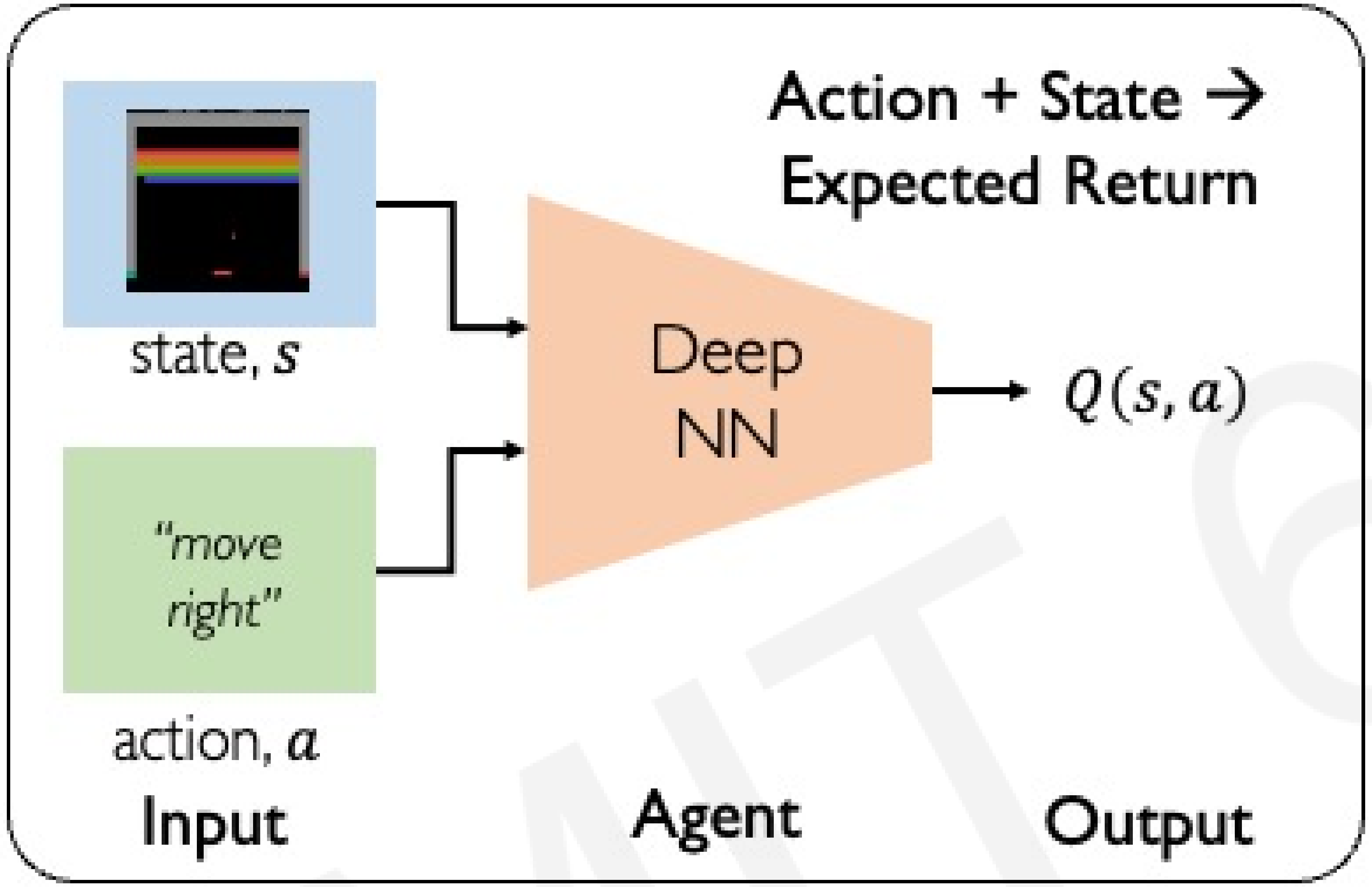
$$\overbrace{\left(r + \gamma \max_{a'} Q(s',a')\right)}^{\text{target}}$$

Take all the best actions → target return

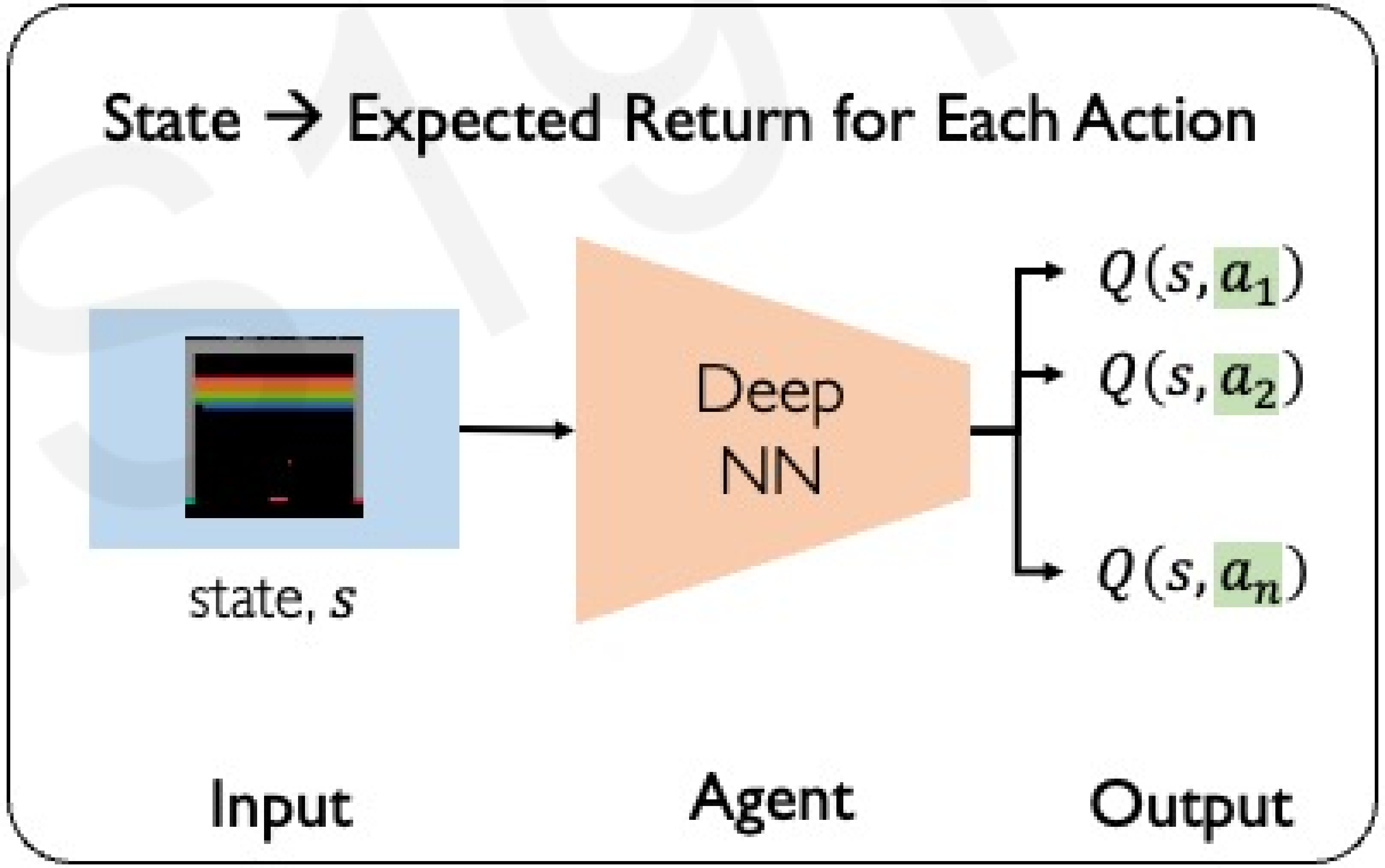
Mnih+ *Nature* 2015.

Deep Q Networks (DQN): Training

How can we use deep neural networks to model Q-functions?



Action + State → Expected Return



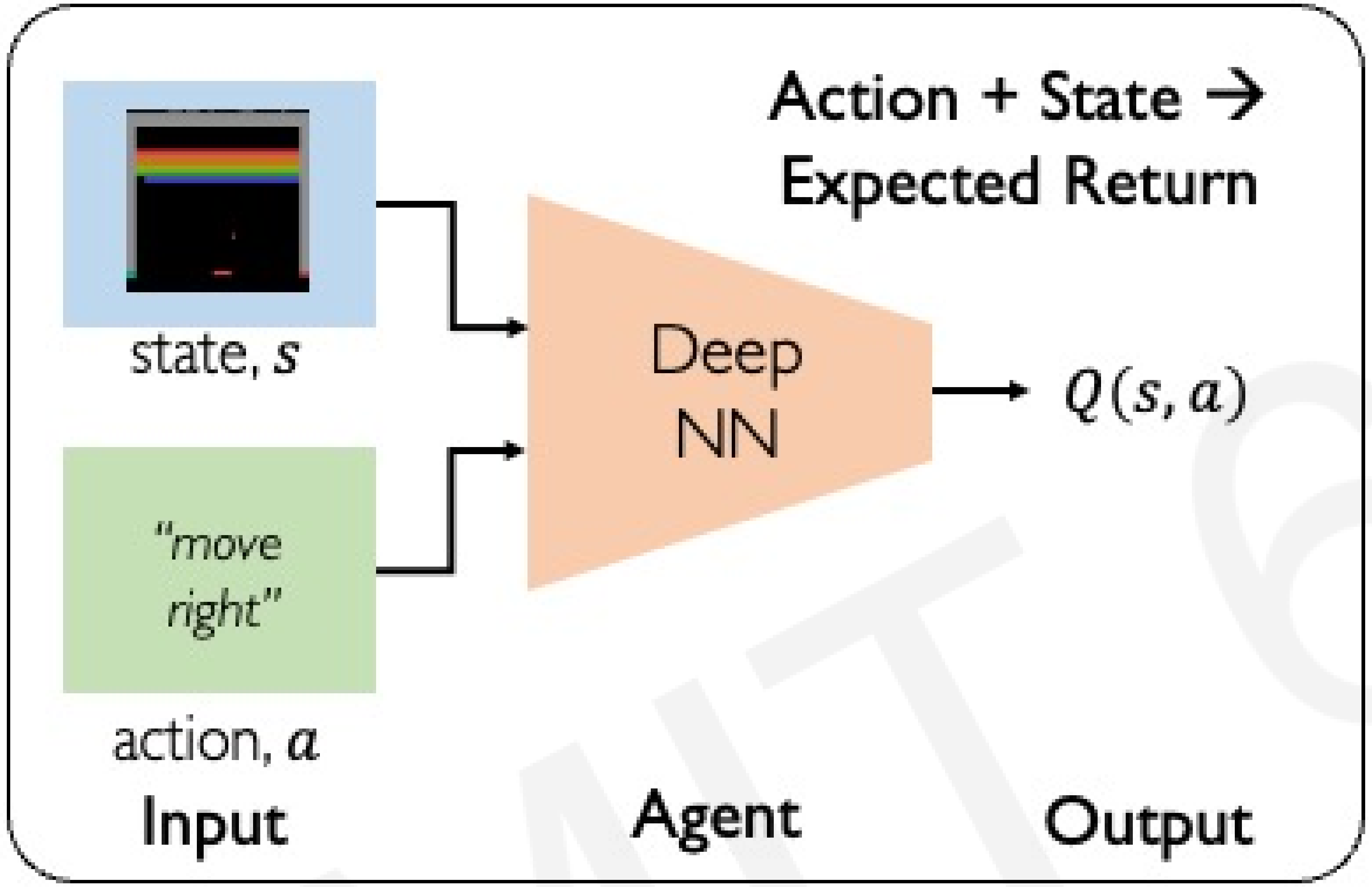
State → Expected Return for Each Action

$$\overbrace{\left(r+\gamma\max_{a'}Q(s',a')\right)}^{\text{target}} \qquad \overbrace{Q(s,a)}^{\text{predicted}}$$

Network prediction

Deep Q Networks (DQN): Training

How can we use deep neural networks to model Q-functions?



Action + State → Expected Return

state, s

"move right"

action, a

Deep NN

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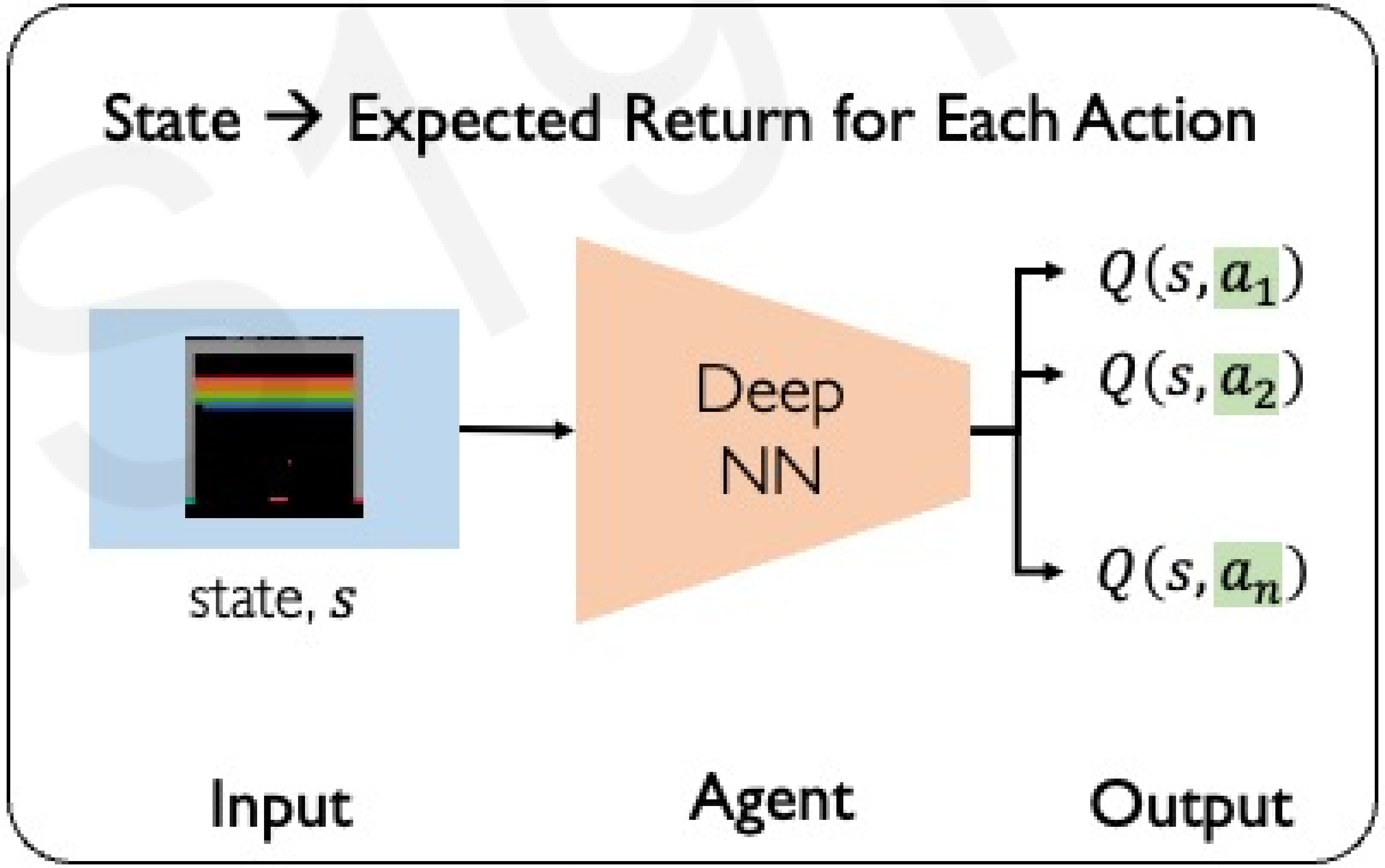
Input **Agent** **Output**



State → Expected Return for Each Action

state, s

Deep NN

$Q(s,a_1)$
$Q(s,a_2)$
$Q(s,a_n)$

Input **Agent** **Output**

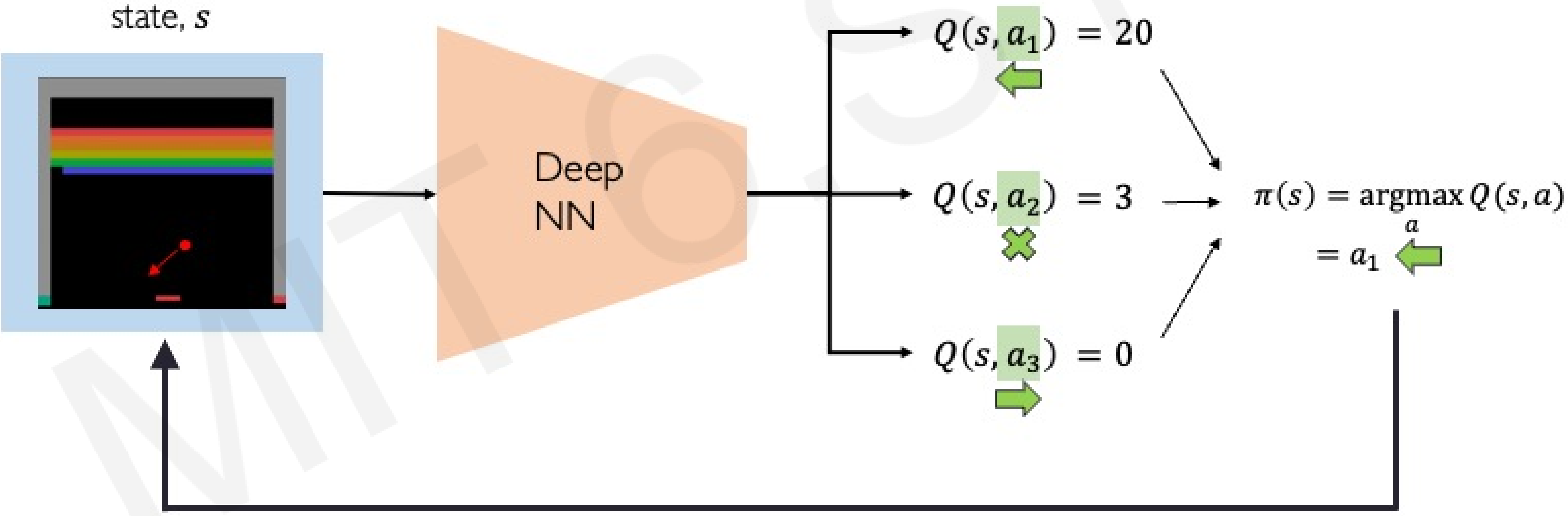
$$\mathcal{L} = \mathbb{E}\left[\left\| \overbrace{\left(r + \gamma \max_{a'} Q(s', a')\right)}^{\text{target}} - \overbrace{Q(s,a)}^{\text{predicted}} \right\|^2\right]$$

Q-Loss

Mnih+ *Nature* 2015. 1/10/24

Deep Q Network Summary

Use NN to learn Q-function and then use to infer the optimal policy, $\pi(s)$



state, s

Deep NN

$Q(s, a_1) = 20$ ⇦

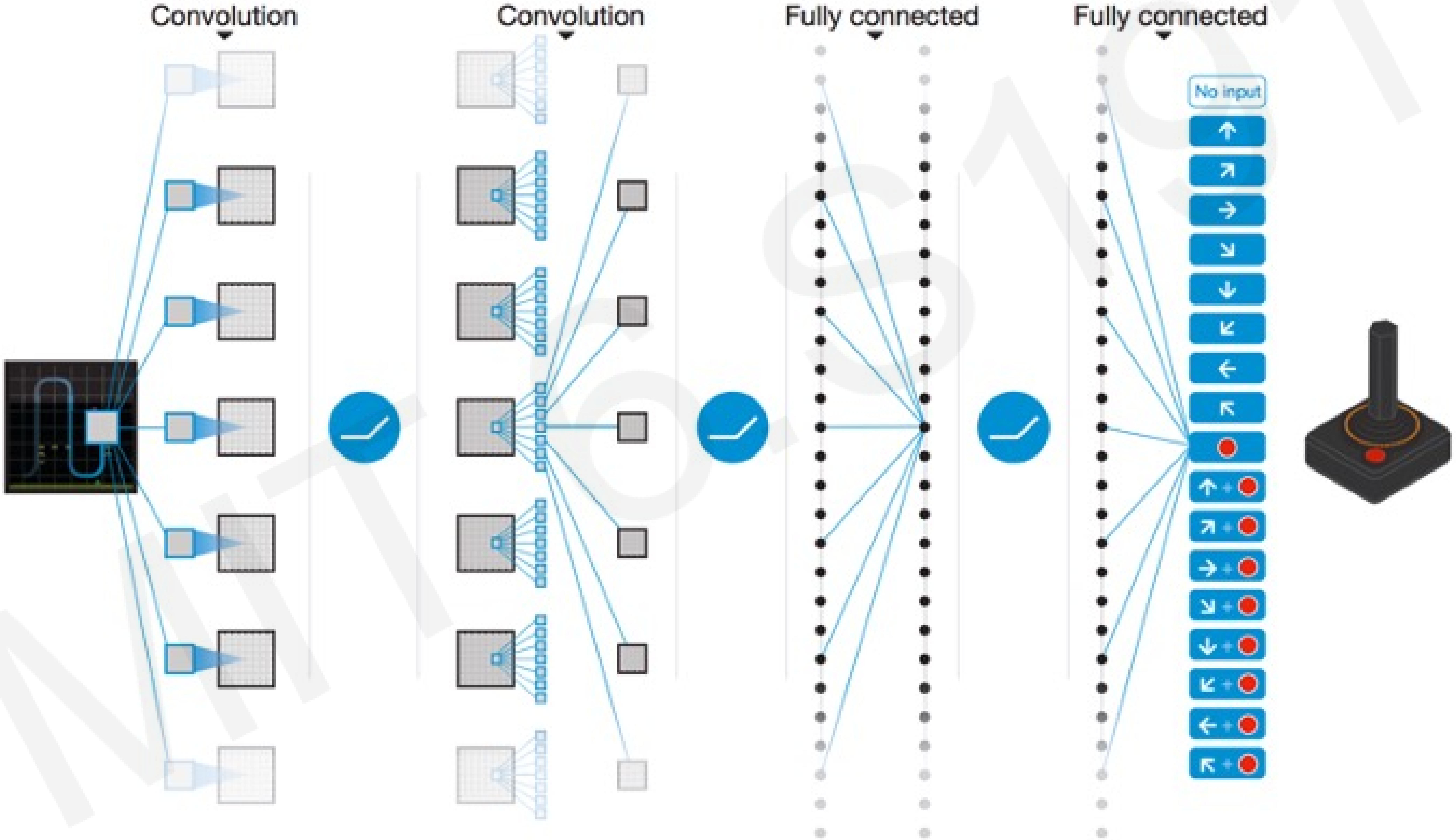
$Q(s, a_2) = 3$ ✖

$Q(s, a_3) = 0$ ⇨

$$\pi(s) = \underset{a}{\operatorname{argmax}}\, Q(s,a) = a_1$$ ⇦

Send action back to environment and receive next state

DQN Atari Results



Mnih+ *Nature* 2015.

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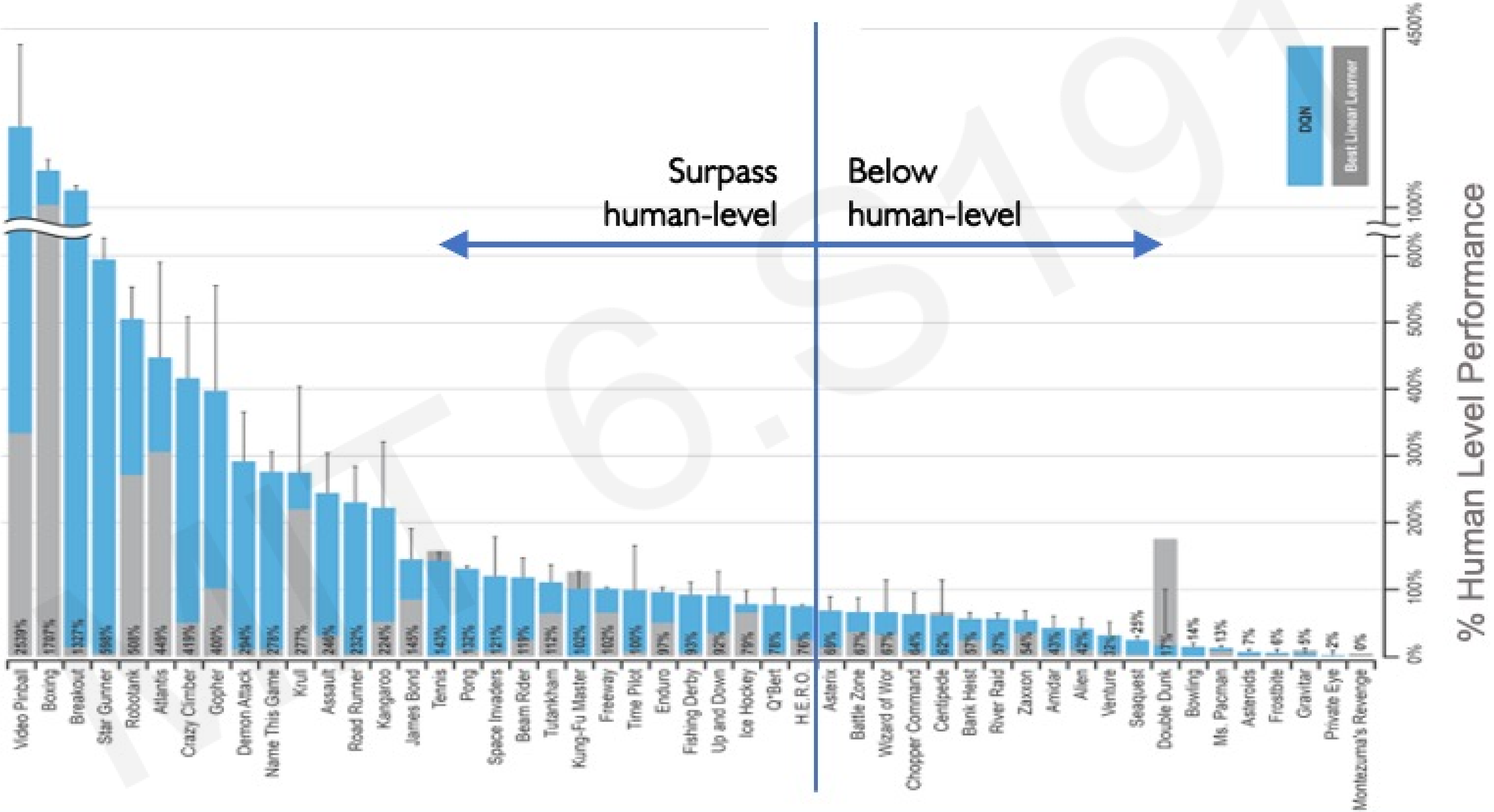


Surpass human-level

Below human-level

Mnih+ *Nature* 2015.

Downsides of Q-learning

Complexity:
- Can model scenarios where the action space is discrete and small
- Cannot handle continuous action spaces

Flexibility:
- Policy is deterministically computed from the Q function by maximizing the reward → cannot learn stochastic policies

To address these, consider a new class of RL training algorithms:
Policy gradient methods

Deep Reinforcement Learning Algorithms

Value Learning

Find $Q(s, a)$

$$a = \underset{a}{\operatorname{argmax}}\, Q(s, a)$$

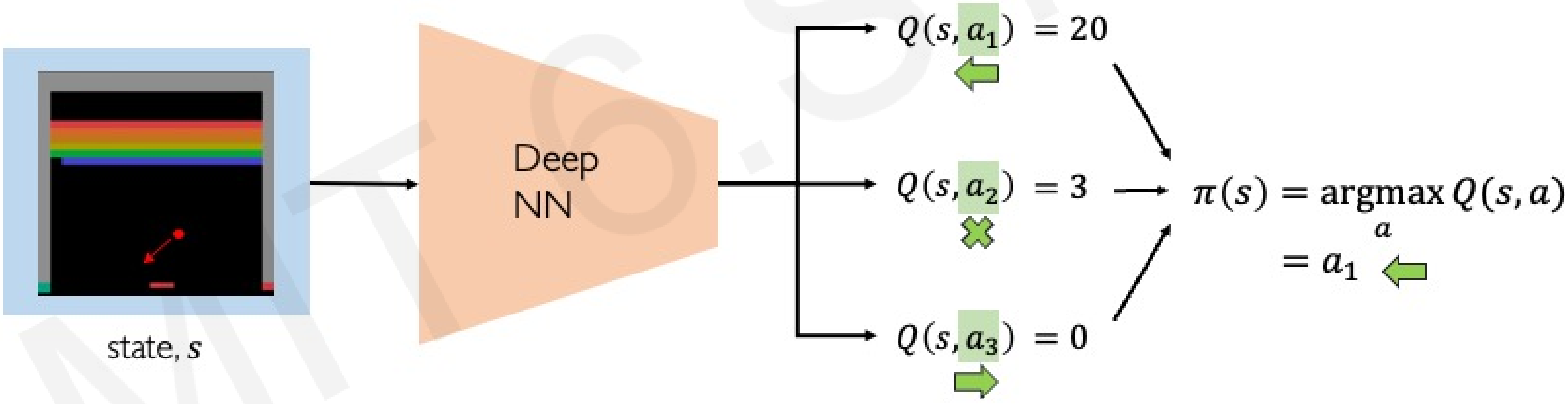
Policy Learning

Find $\pi(s)$

Sample $a \sim \pi(s)$

Deep Q Networks (DQN)

DQN: Approximate Q-function and use to infer the optimal policy, $\pi(s)$



state, s

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$Q(s, a_2) = 3$ ✖

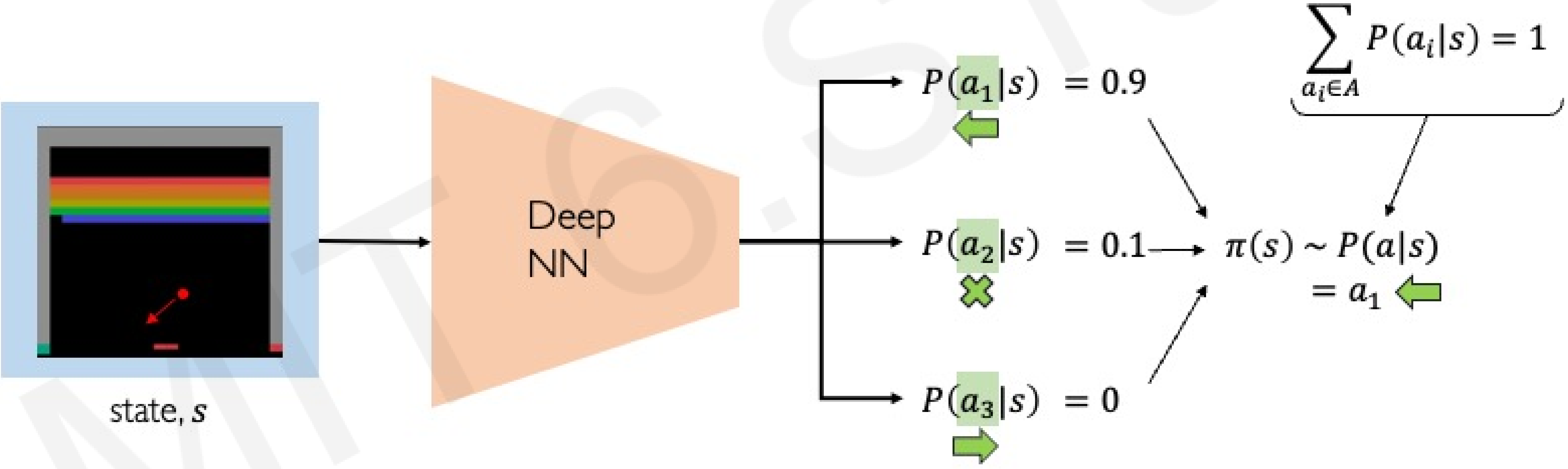
$Q(s, a_3) = 0$ ⇨

$$\pi(s) = \underset{a}{\operatorname{argmax}}\, Q(s,a)$$
$$= a_1$$ ⇦

Policy Gradient (PG): Key Idea

DQN: Approximate Q-function and use to infer the optimal policy, $\pi(s)$

Policy Gradient: Directly optimize the policy $\pi(s)$



state, s

Deep NN

$P(a_1|s) = 0.9$ ⇦

$P(a_2|s) = 0.1$ ✖

$P(a_3|s) = 0$ ⇨

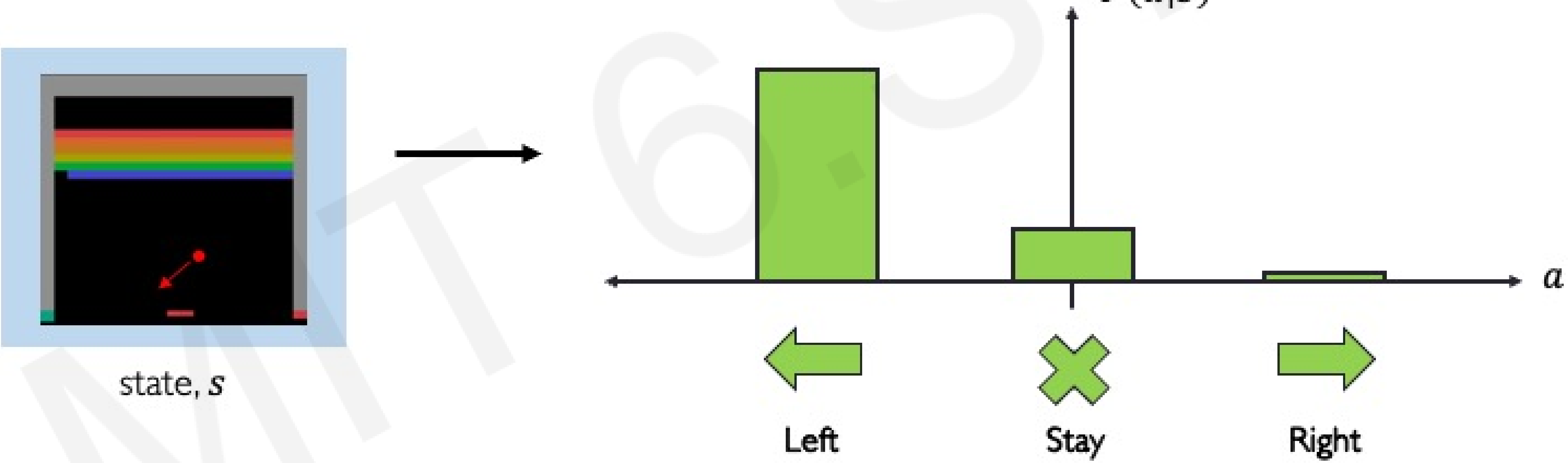
$$\underbrace{\sum_{a_i \in A} P(a_i|s) = 1}$$

$$\pi(s) \sim P(a|s) = a_1$$ ⇦

🤔 What are some advantages of this formulation?

Discrete vs Continuous Action Spaces

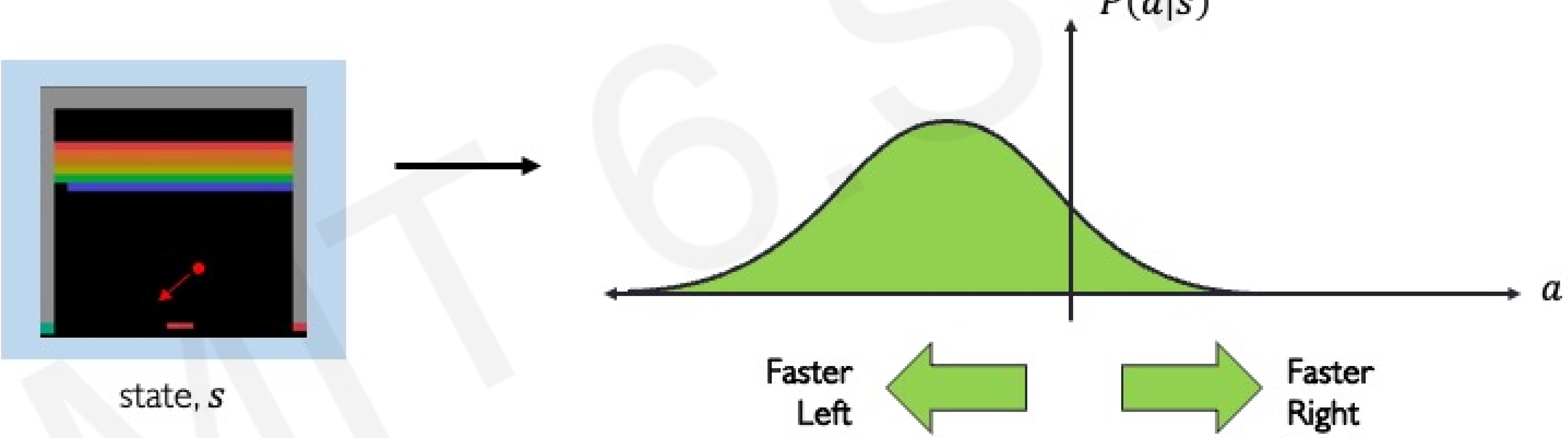
Discrete action space: which direction should I move? ⇦ ✖ ⇨



Discrete vs Continuous Action Spaces

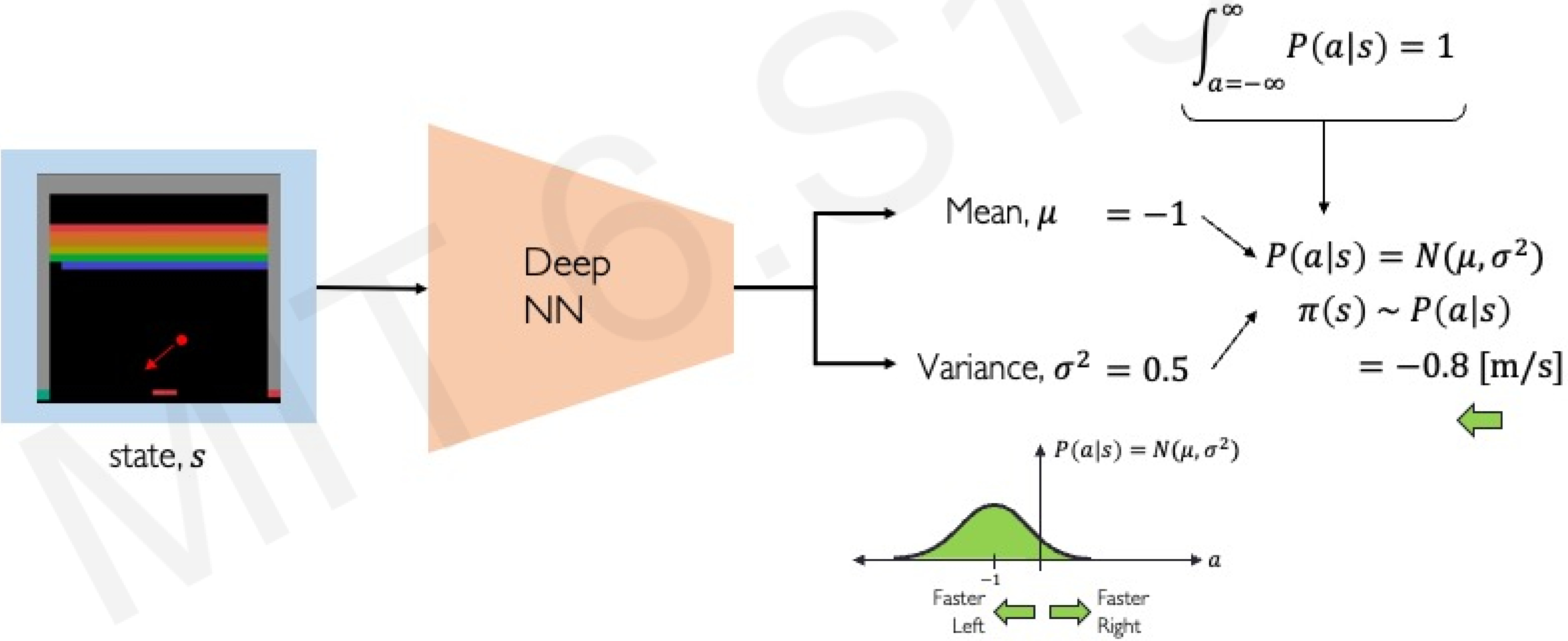
Discrete action space: which direction should I move? ⇦ ✖ ⇨

Continuous action space: how fast should I move? ⇦ 0.7 m/s



Policy Gradient (PG): Key Idea

Policy Gradient: Enables modeling of continuous action space



state, s

Deep NN

Mean, $\mu = -1$

Variance, $\sigma^2 = 0.5$

$$\underbrace{\int_{a=-\infty}^{\infty} P(a|s) = 1}$$

$$P(a|s) = N(\mu, \sigma^2)$$

$$\pi(s) \sim P(a|s)$$

$$= -0.8 \text{ [m/s]}$$

Training Policy Gradients: Case Study

Reinforcement Learning Loop:

Case Study – Self-Driving Cars

Agent: vehicle

State: camera, lidar, etc

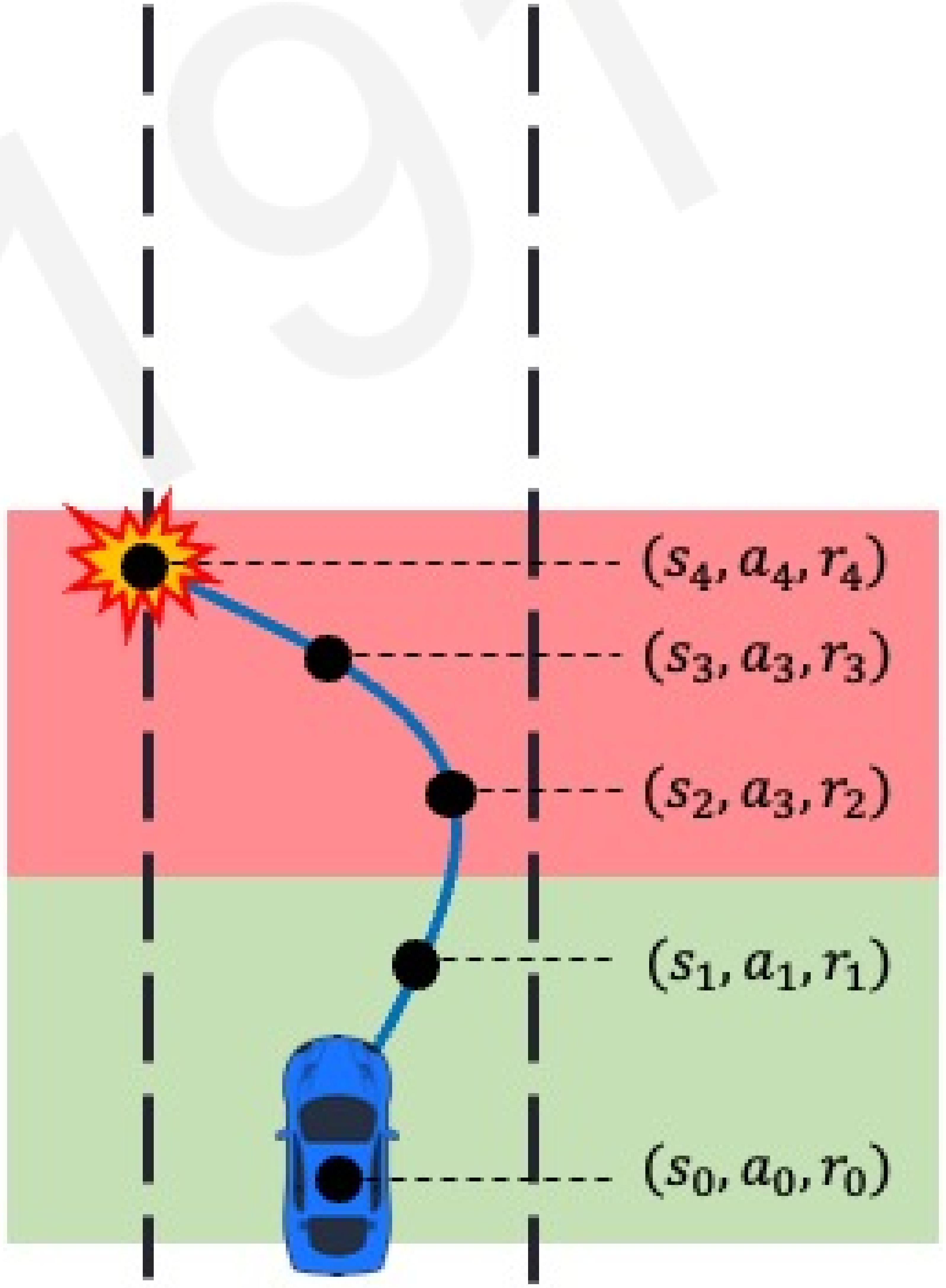
Action: steering wheel angle

Reward: distance traveled

Training Policy Gradients

Training Algorithm

1. Initialize the agent
2. Run a policy until termination
3. Record all states, actions, rewards
4. Decrease probability of actions that resulted in low reward
5. Increase probability of actions that resulted in high reward



Williams *Reinforcement Learning* 1992.

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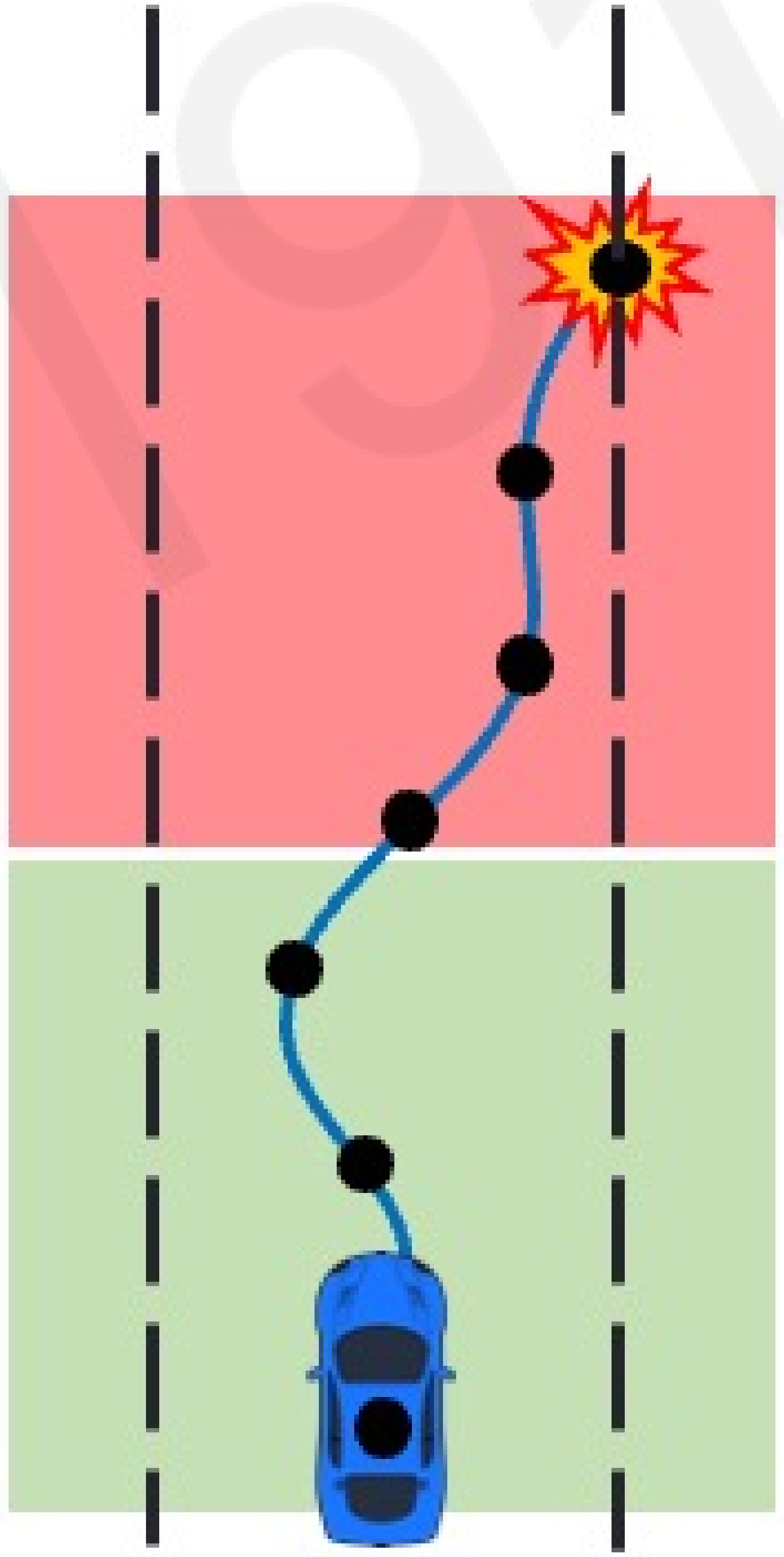

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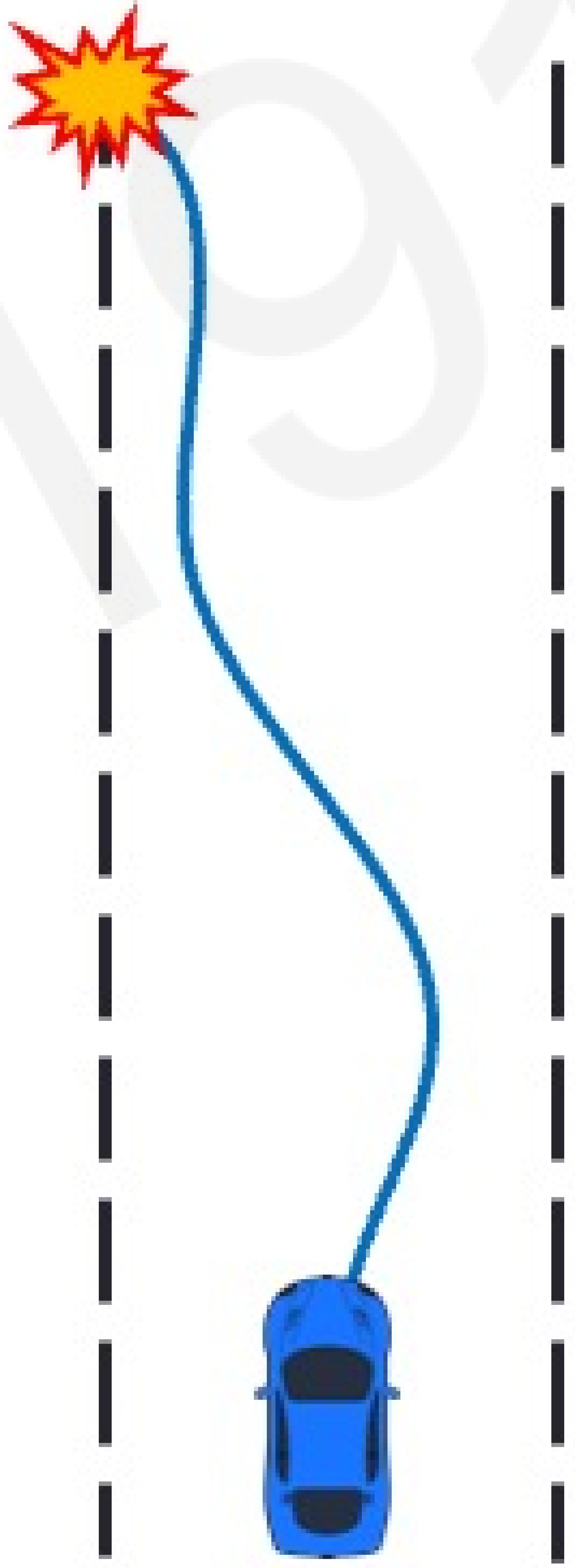

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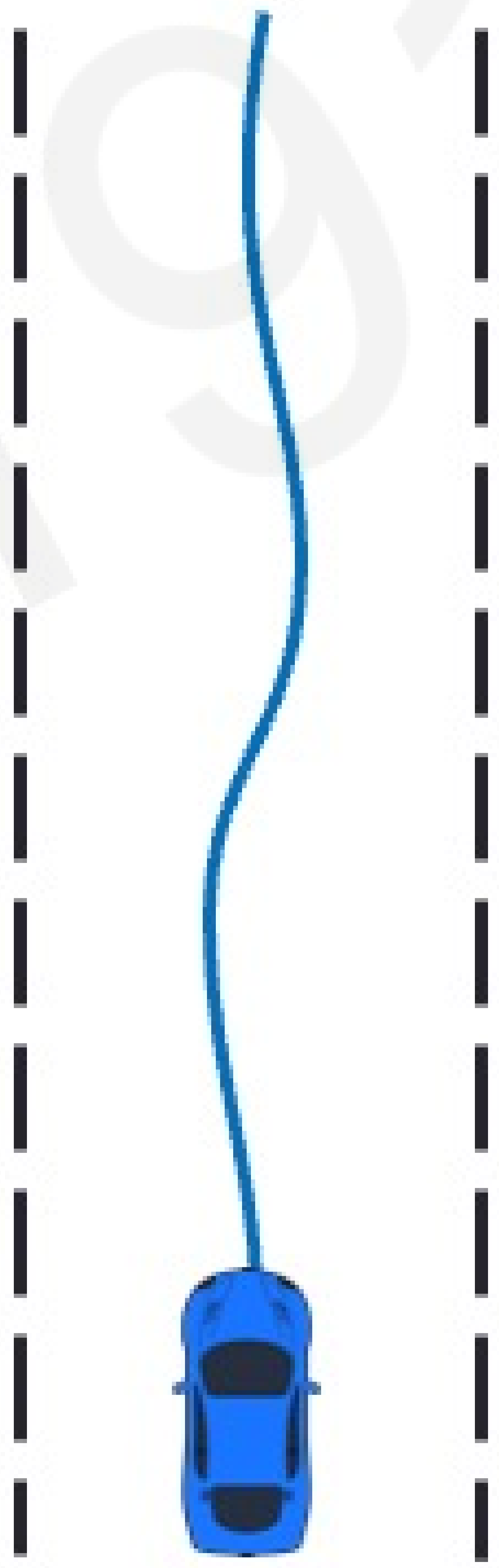

Training Policy Gradients

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$$\mathbf{loss} = -\overbrace{\log \mathrm{P}(a_t|s_t)}^{\text{log-likelihood of action}}\,\underbrace{R_t}_{\text{reward}}$$

Gradient descent update:

$$w' = w - \nabla\,\mathbf{loss}$$
$$w' = w + \underbrace{\boxed{\nabla \log \mathrm{P}(a_t|s_t)\,R_t}}_{\text{Policy gradient!}}$$

Williams *Reinforcement Learning* 1992. 1/10/24

Reinforcement Learning in Real Life

Training Algorithm

1. Initialize the agent
2. Run a policy until termination
3. Record all states, actions, rewards
4. Decrease probability of actions that resulted in low reward
5. Increase probability of actions that resulted in high reward

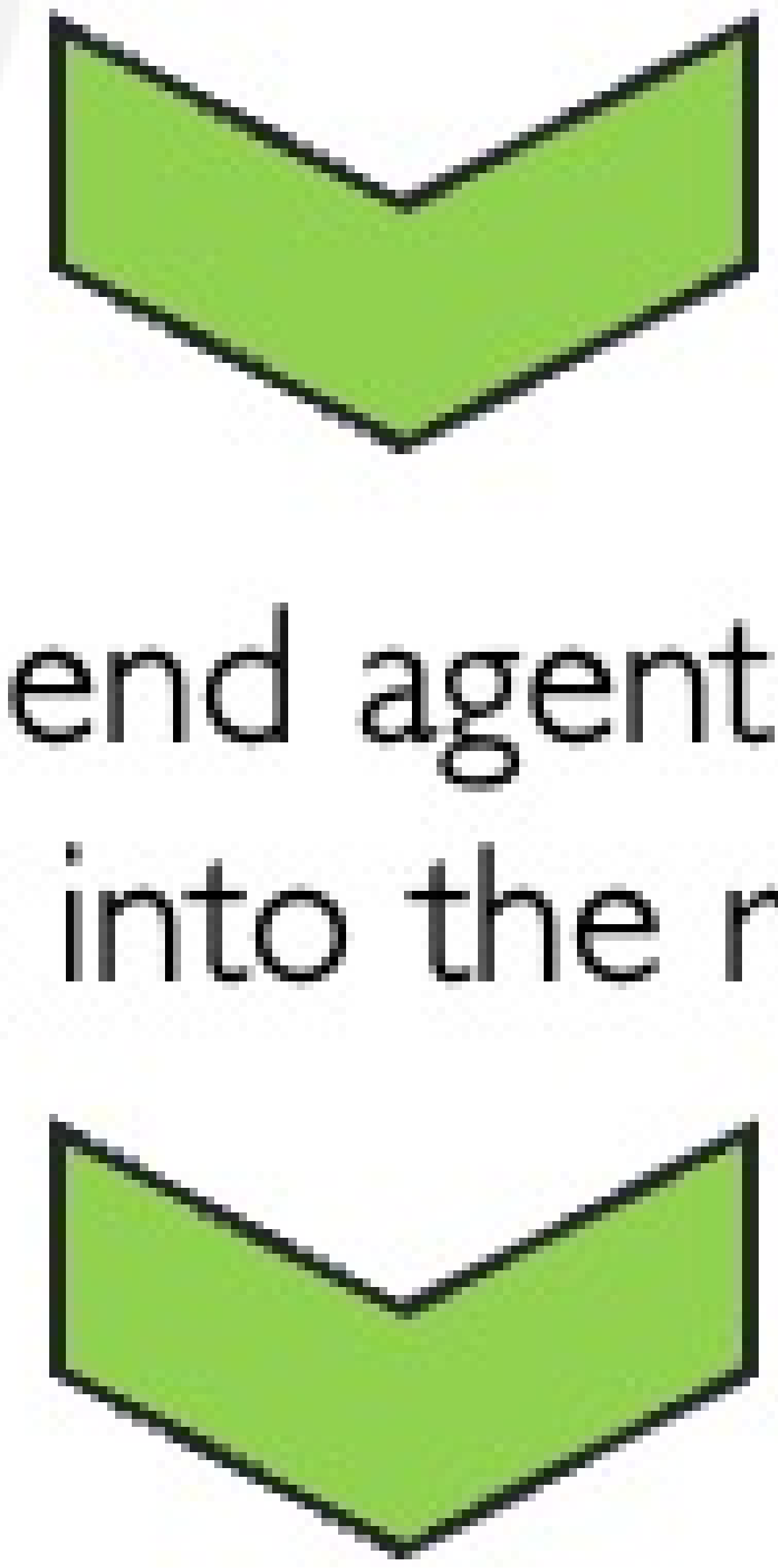
Amini+ *IEEE RA-L* 2020.

Data-driven Simulation for Autonomous Vehicles

VISTA: Photorealistic and high-fidelity simulator for training and testing self-driving cars

Amini+ *IEEE RA-L* 2020.

Deploying End-to-End RL for Autonomous Vehicles



Policy Gradient RL agent trained entirely within VISTA simulator

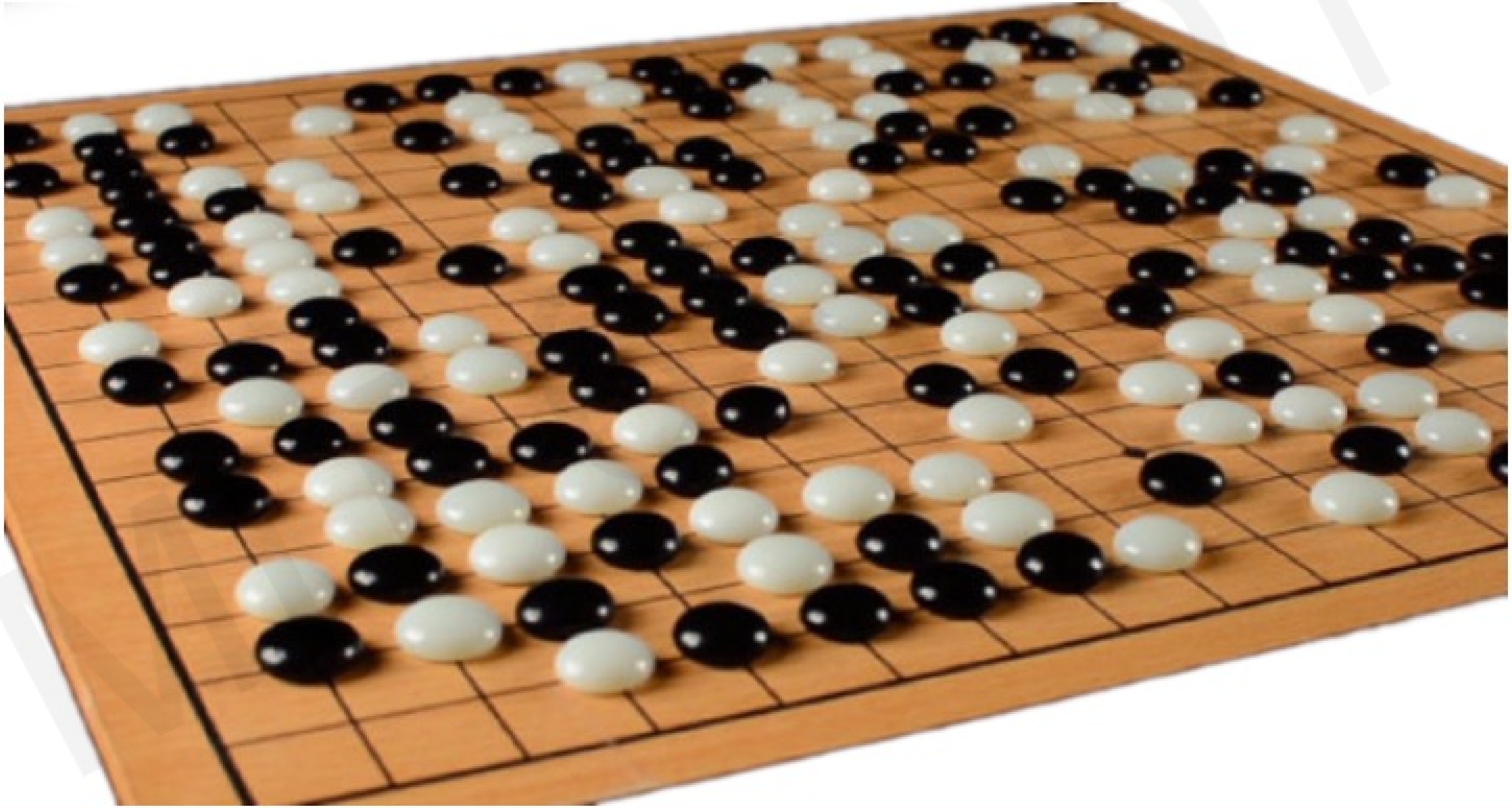
End-to-end agent directly deployed into the real-world

First full-scale autonomous vehicle trained using RL entirely in simulation and deployed in real life!

Amini+ *IEEE RA-L* 2020.

Deep Reinforcement Learning Applications

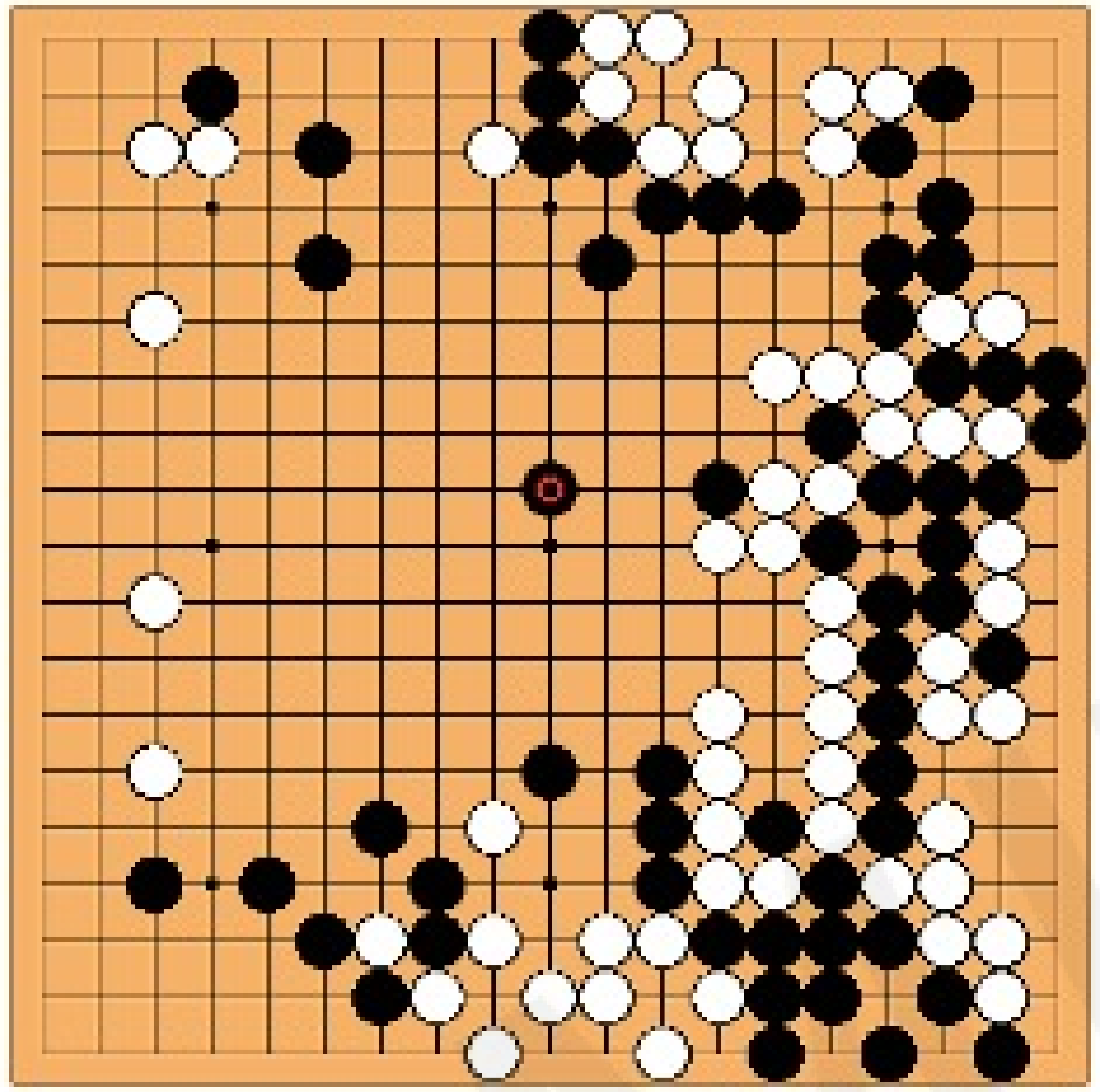
Reinforcement Learning and the Game of Go



Silver+ *Science* 2016; Silver+ *Nature* 2017.

The Game of Go

Aim: Get more board territory than your opponent.



Board Size n x n	Positions 3^{n^2}	% Legal	Legal Positions
1×1	3	33.33%	1
2×2	81	70.37%	57
3×3	19,683	64.40%	12,675
4×4	43,046,721	56.49%	24,318,165
5×5	847,288,609,443	48.90%	414,295,148,741
9×9	$4.434264882 \times 10^{38}$	23.44%	$1.039191487910 \times 10^{38}$
13×13	$4.300233593 \times 10^{80}$	8.66%	$3.72497923077 \times 10^{79}$
19×19	$1.740896506 \times 10^{172}$	1.20%	$2.08168199382 \times 10^{170}$

Greater number of legal board positions than atoms in the universe.

Source: Wikipedia.

AlphaGo Beats Top Human Player at Go

Silver+ Science 2016.

AlphaGo Beats Top Human Player at Go

1) Initial training: human data

Silver+ Science 2016.

AlphaGo Beats Top Human Player at Go

Human expert positions

Classification

Supervised Learning policy network

Self Play

RL policy network

Self Play

Self-play data

Regression

Value network

1) Initial training: human data

2) Self-play and reinforcement learning
→ super-human performance

Silver+ Science 2016.

AlphaGo Beats Top Human Player at Go

1) Initial training: human data

2) Self-play and reinforcement learning
→ super-human performance

3) "Intuition" about board state

Silver+ Science 2016.

AlphaZero: RL from Self-Play

Silver+ Science 2018.

Deep Reinforcement Learning: Summary

Foundations

- Agents acting in environment
- State-action pairs → maximize future rewards
- Discounting

Q-Learning

- Q function: expected total reward given **s, a**
- Policy determined by selecting action that maximizes Q function

Policy Gradients

- Learn and optimize the policy directly
- Applicable to continuous action spaces

T-Shirts Coming Tomorrow!

SYLLABUS: **bit.ly/6s191-syllabus**

1. Project sign-ups due **TODAY 1/10 11:00pm ET**

2. Lab competitions and prizes!
DEADLINE: Thursday 1/11 11:00pm ET

3. **Project and lab submission links on syllabus!**

4. **RESUME DROP** to sponsoring companies!
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