Deep Generative Modeling

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MIT Introduction to Deep Learning

January 7, 2025

Which face is real?

A

B

C

Supervised vs unsupervised learning

Supervised Learning

Data: (x, y)
x is data, y is label

Goal: Learn function to map
$$x \rightarrow y$$

Examples: Classification, regression, object detection, semantic segmentation, etc.

Unsupervised Learning

Data: x
x is data, no labels!

Goal: Learn some *hidden* or *underlying structure* of the data

Examples: Clustering, feature or dimensionality reduction, etc.

Supervised vs unsupervised learning

Supervised Learning

Data: (x, y)
x is data, y is label

Goal: Learn function to map
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Unsupervised Learning

Data: x
x is data, no labels!

Goal: Learn the *hidden* or *underlying structure* of the data

Examples: Clustering, feature or dimensionality reduction, etc.

Generative modeling

Goal: Take as input training samples from some distribution and learn a model that represents that distribution

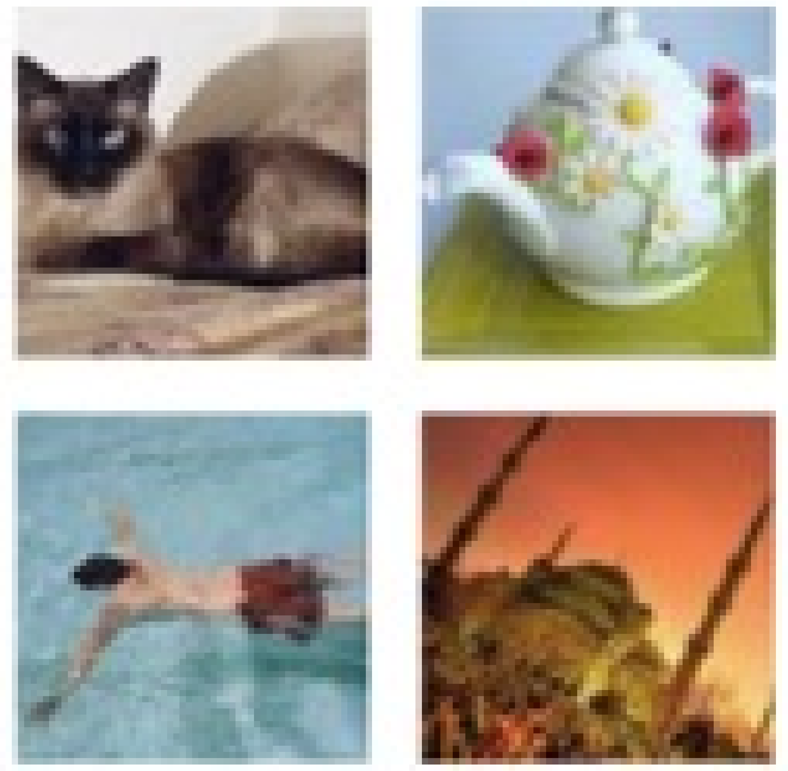
Density Estimation

samples

Sample Generation

Input samples

Training data $\sim P_{data}(x)$



Generated samples

Generated $\sim P_{model}(x)$

How can we learn $P_{model}(x)$ similar to $P_{data}(x)$?

Why generative models? Debiasing

Capable of uncovering **underlying features** in a dataset

Homogeneous skin color, pose

vs

Diverse skin color, pose, illumination

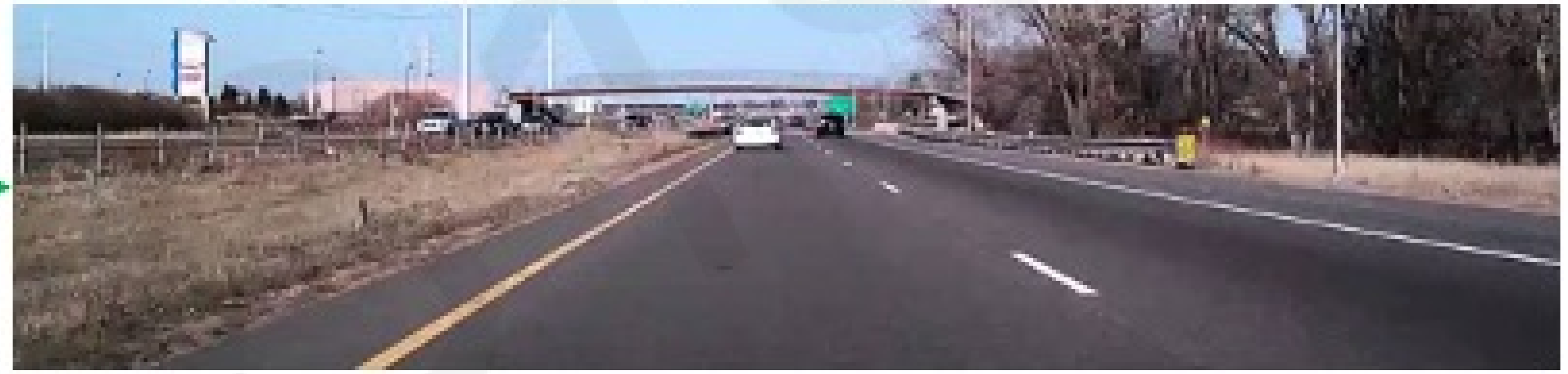
How can we use this information to create fair and representative datasets?

Software Lab!

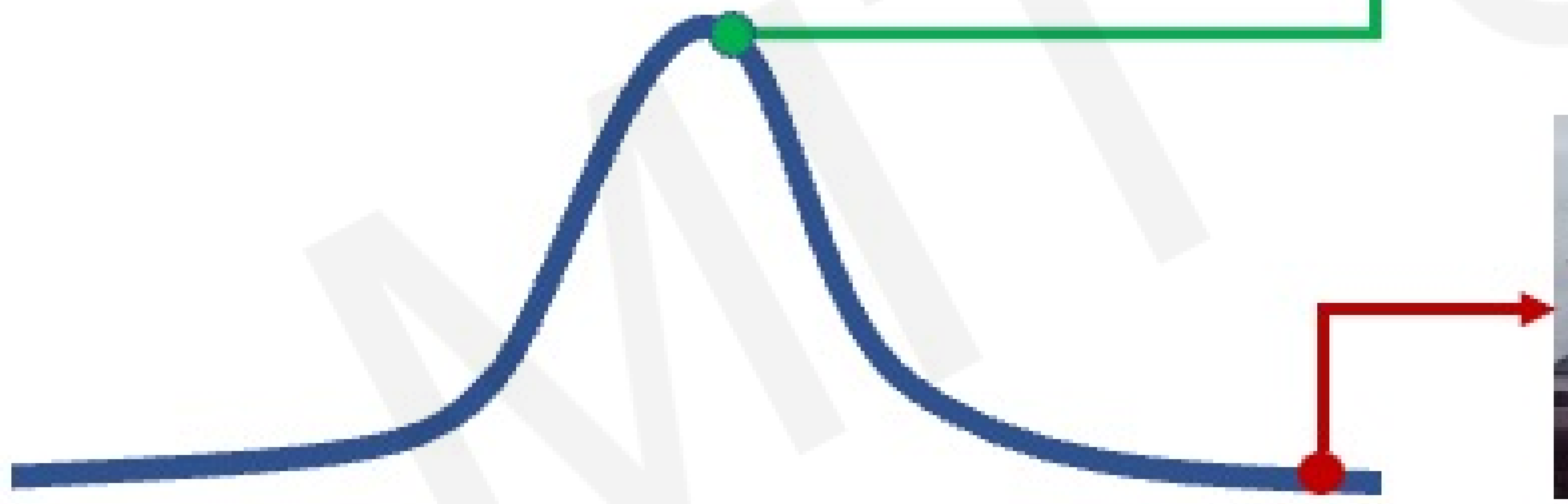
Why generative models? Outlier detection

- **Problem:** How can we detect when we encounter something new or rare?
- **Strategy:** Leverage generative models, detect outliers in the distribution
- Use outliers during training to improve even more!

95% of Driving Data:
(1) sunny, (2) highway, (3) straight road

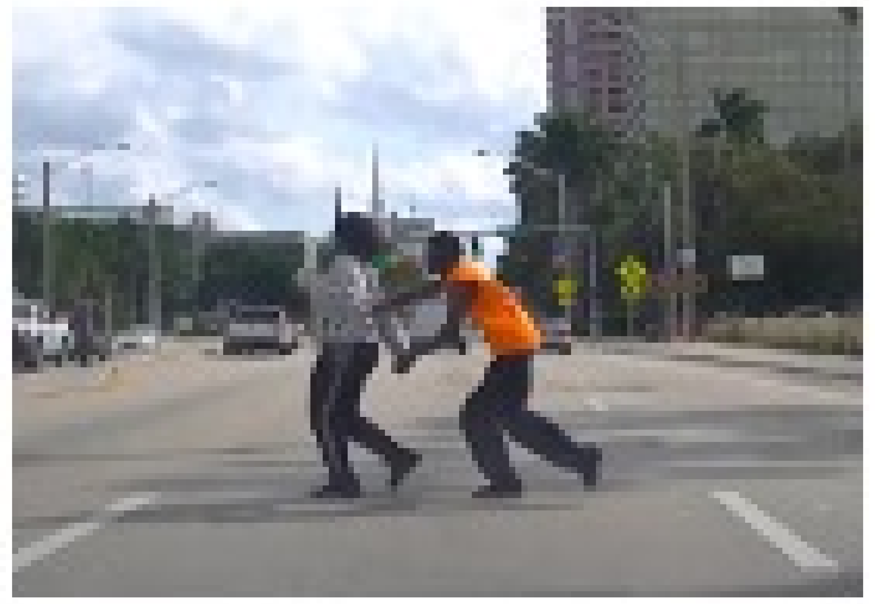


Detect outliers to avoid unpredictable behavior when training



Edge Cases

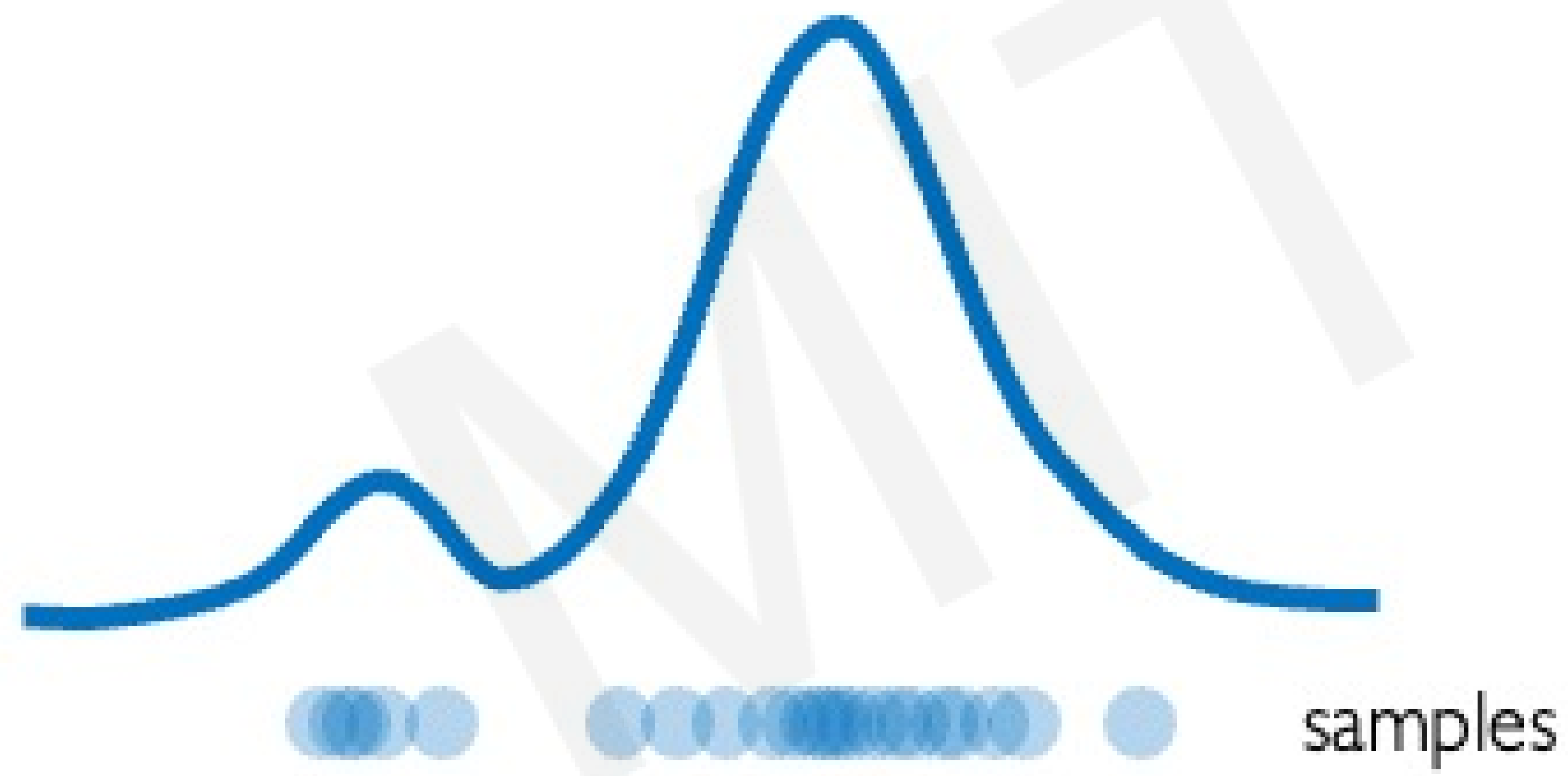
Harsh Weather



Pedestrians

Why generative models? Sample generation

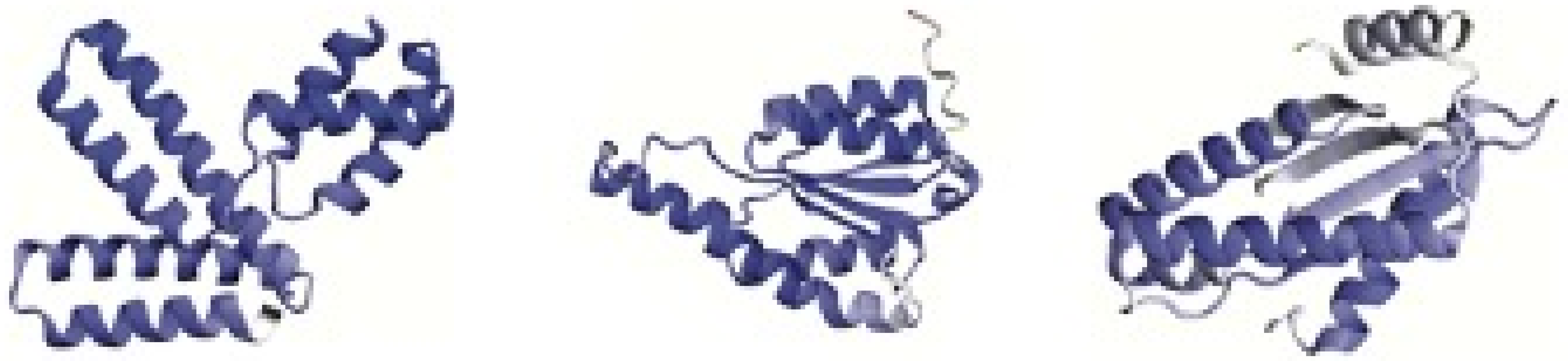
- **Generative models learn probability distributions**
- **Sampling** from that distribution → new data instances
- **Backbone of Generative AI:** generate language, images, and more



Natural Language

6.S191 Guest Lectures!

Images & Videos

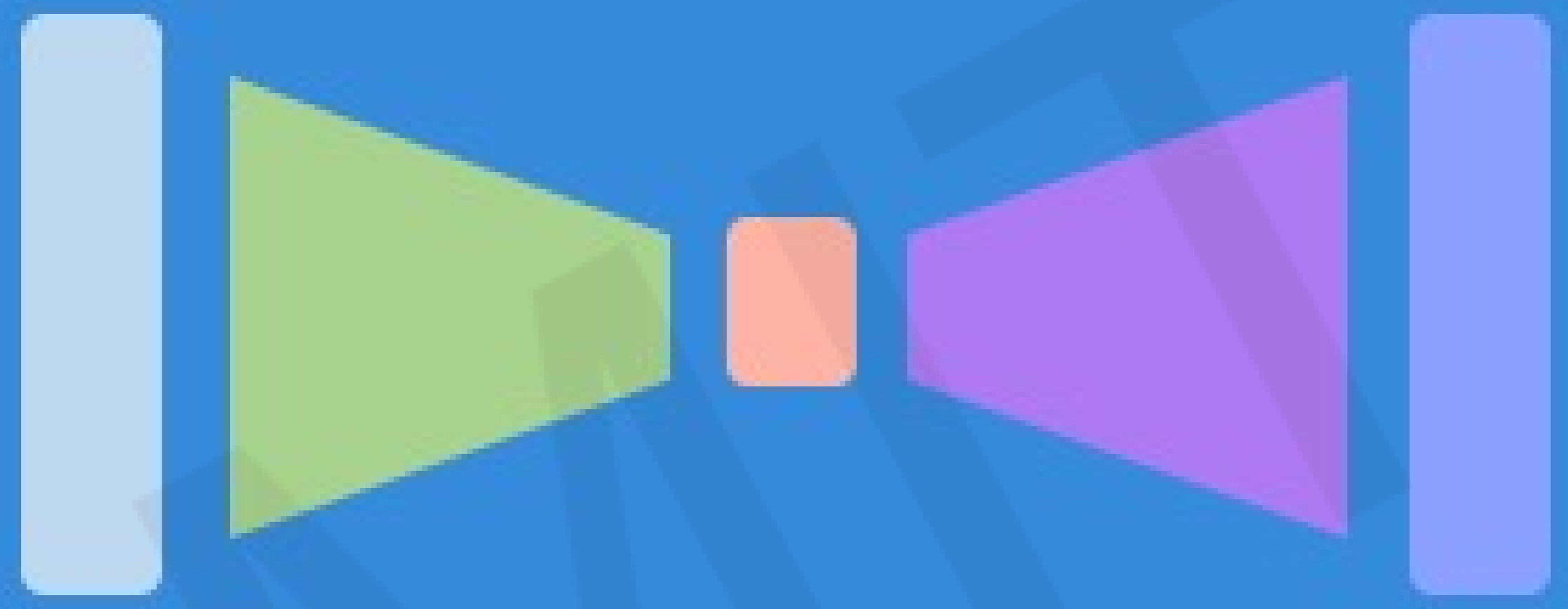


Biology

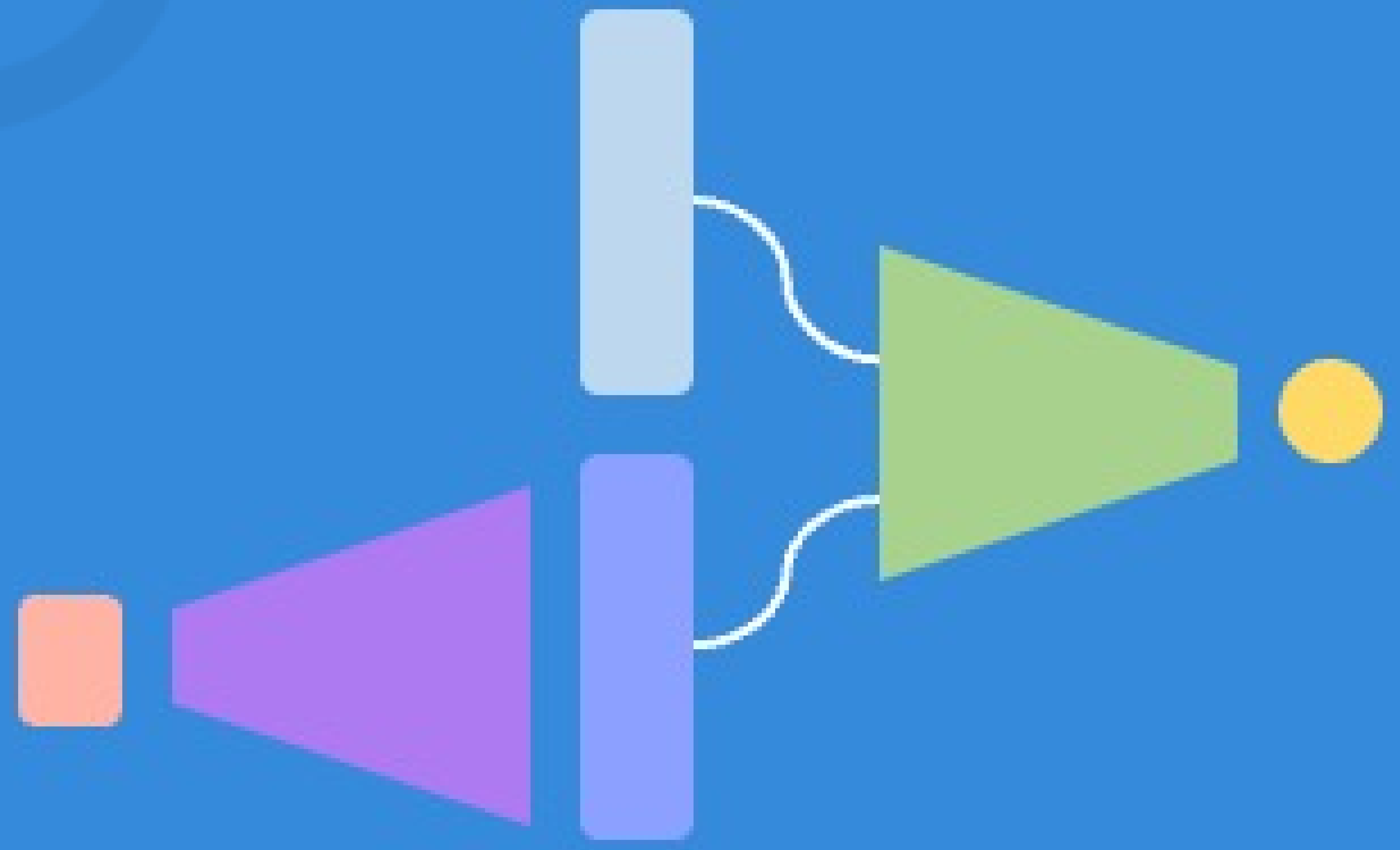
6.S191 Guest Lecture!

Latent variable models

Autoencoders and Variational Autoencoders (VAEs)



Generative Adversarial Networks (GANs)



What is a latent variable?

Myth of the Cave

What is a latent variable?

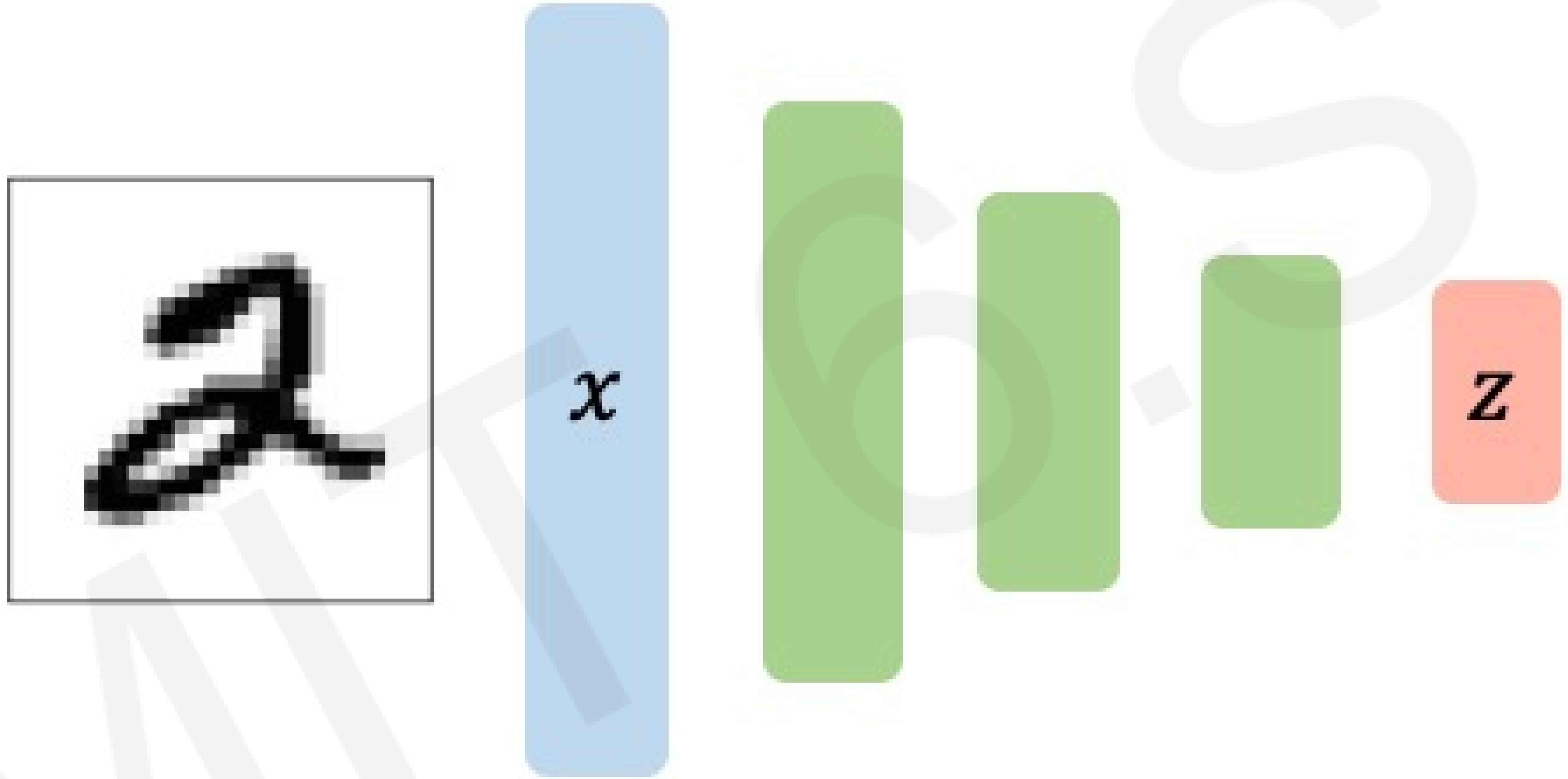
Can we learn the **true explanatory factors**, e.g. latent variables, from only observed data?

Plato, *Republic.*

Autoencoders

Autoencoders: background

Unsupervised approach for learning a **lower-dimensional** feature representation from unlabeled training data



Why do we care about a low-dimensional z?

"Encoder" learns mapping from the data, x, to a low-dimensional latent space, z

Autoencoders: background

How can we learn this latent space?
Train the model to use these features to **reconstruct the original data**

"Decoder" learns mapping back from latent space, z,
to a reconstructed observation, $\hat{x}$

Autoencoders: background

How can we learn this latent space?
Train the model to use these features to **reconstruct the original data**

$$\mathcal{L}(x, \hat{x}) = ||x - \hat{x}||^2$$

Loss function doesn't use any labels!!

Autoencoders: background

How can we learn this latent space?

Train the model to use these features to **reconstruct the original data**

$$\mathcal{L}(x, \hat{x}) = \|x - \hat{x}\|^2$$

Loss function doesn't use any labels!!

Dimensionality of latent space → reconstruction quality

Autoencoding is a form of compression!
Smaller latent space will force a larger training bottleneck

2D latent space

5D latent space

Ground Truth

Autoencoders for representation learning

Bottleneck hidden layer forces network to learn a compressed latent representation

Reconstruction loss forces the latent representation to capture (or encode) as much "information" about the data as possible

Autoencoding = **Auto**matically **encoding** data; "Auto" = **self**-encoding

Variational Autoencoders (VAEs)

Traditional autoencoders

VAEs: key difference with traditional autoencoder

VAEs: key difference with traditional autoencoder

Variational autoencoders are a probabilistic twist on autoencoders!

Sample from the mean and standard deviation to compute latent sample

VAE optimization

Encoder computes: $q_{\phi}(\mathrm{z}|x)$ Decoder computes: $p_{\theta}(\mathrm{x}|z)$

VAE optimization

$$\mathcal{L}(\phi, \theta, x) = \text{(reconstruction loss)} + \text{(regularization term)}$$

VAE optimization

Encoder computes: $q_{\phi}(z|x)$

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VAE optimization

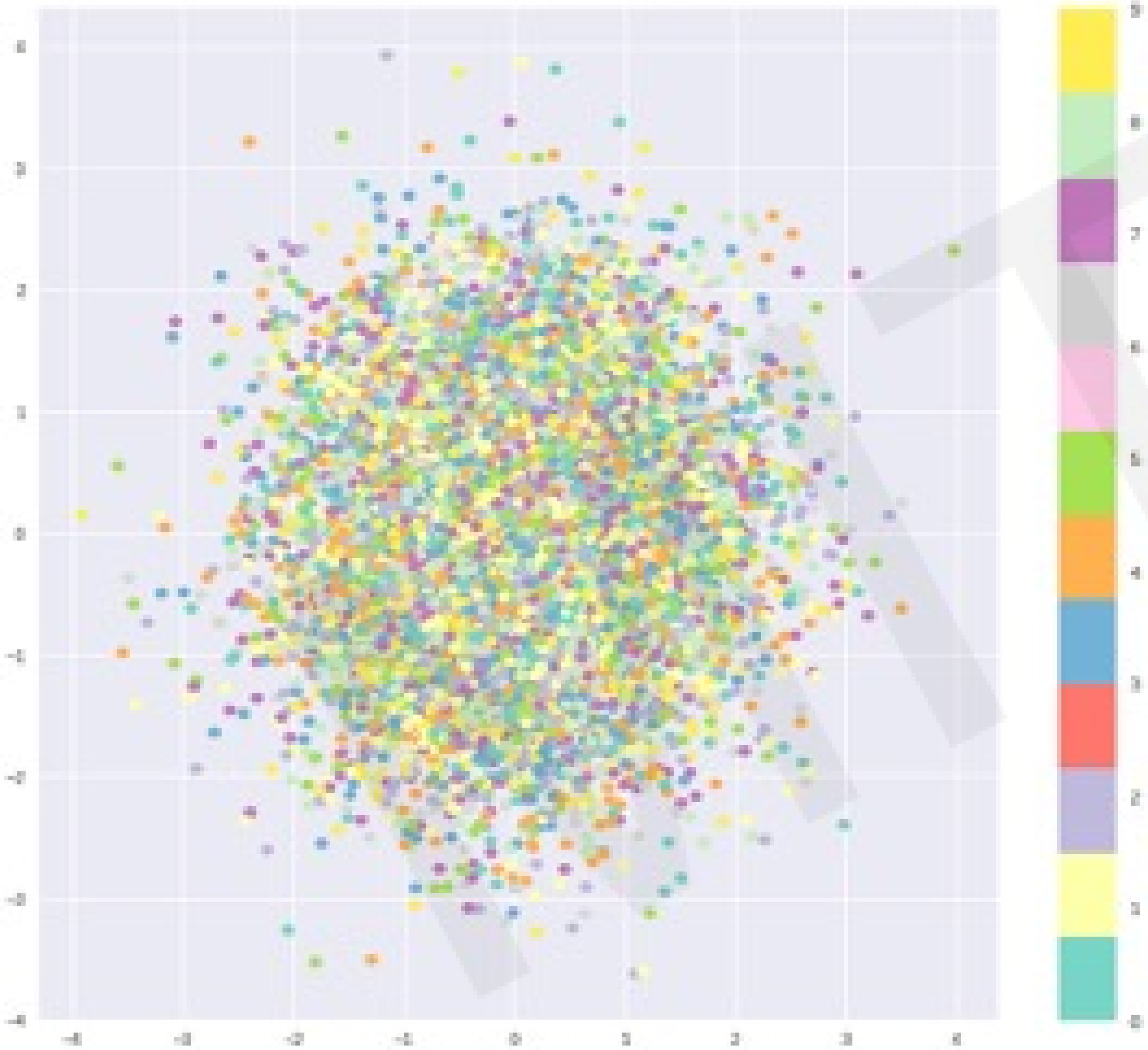
$$\mathcal{L}(\phi, \theta, x) = (\text{reconstruction loss}) + \boxed{(\text{regularization term})}$$

Priors on the latent distribution

$$D\left(q_{\phi}(z|x) \parallel p(z)\right)$$

Inferred latent distribution

Fixed prior on latent distribution



Common choice of prior – Normal Gaussian:

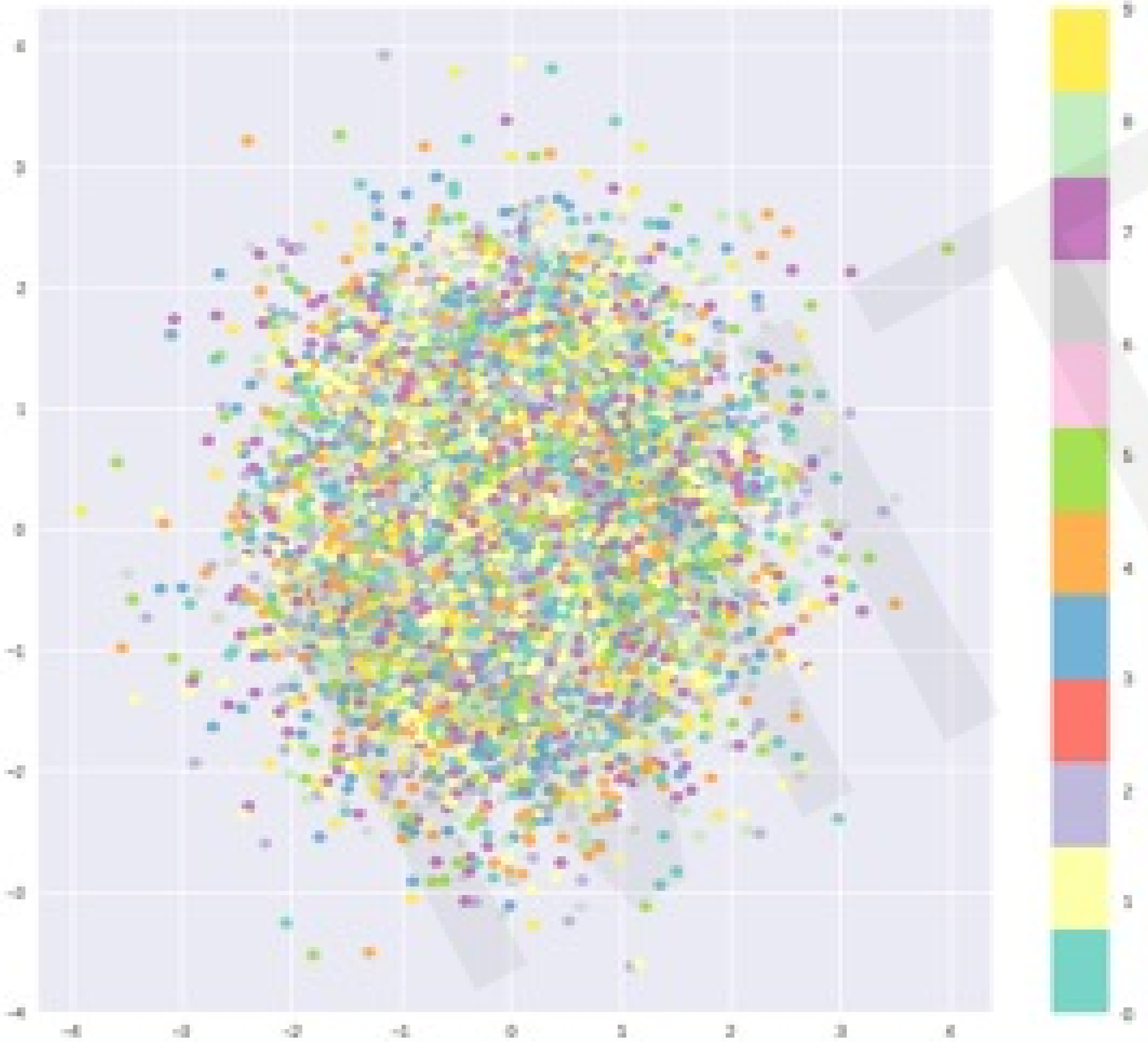
$$p(z) = \mathcal{N}(\mu = 0, \sigma^2 = 1)$$

- Encourages encodings to distribute encodings evenly around the center of the latent space
- Penalize the network when it tries to "cheat" by clustering points in specific regions (i.e., by memorizing the data)

Priors on the latent distribution

$$D\left(q_\phi(\mathrm{z}|x) \parallel p(z)\right)$$
$$= -\frac{1}{2}\sum_{j=0}^{k-1}(\sigma_j + \mu_j^2 - 1 - \log\sigma_j)$$

KL-divergence between the two distributions



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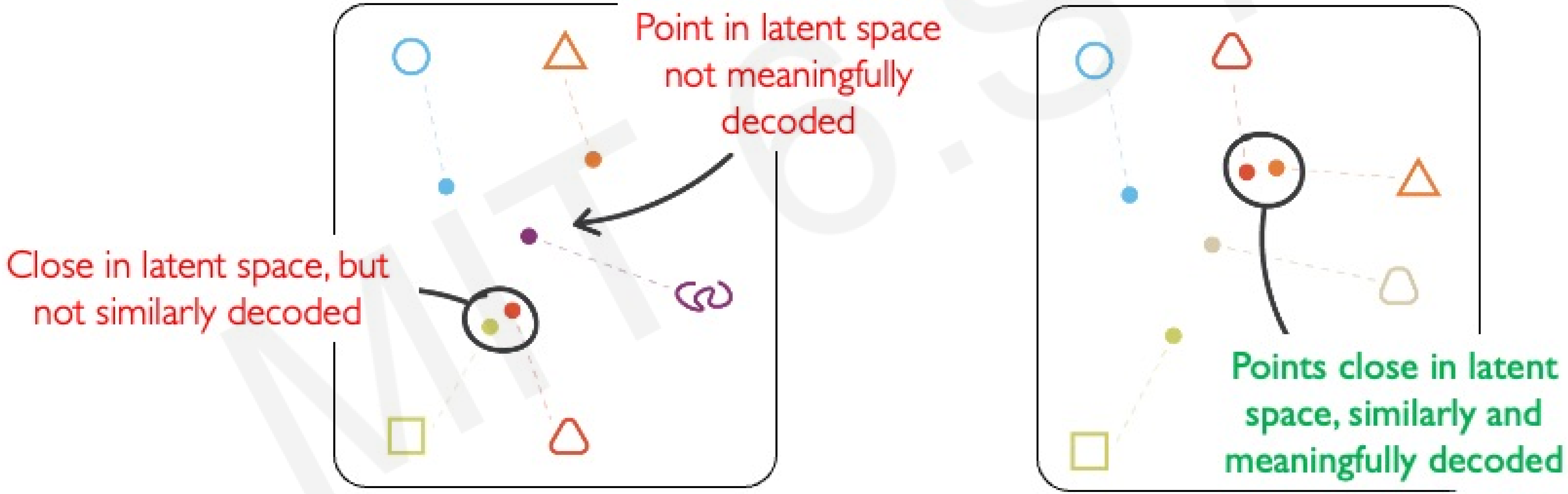
Intuition on regularization and the Normal prior

What properties do we want to achieve from regularization? 🤔

1. **Continuity:** points that are close in latent space → similar content after decoding
2. **Completeness:** sampling from latent space → "meaningful" content after decoding



Not regularized **Regularized**

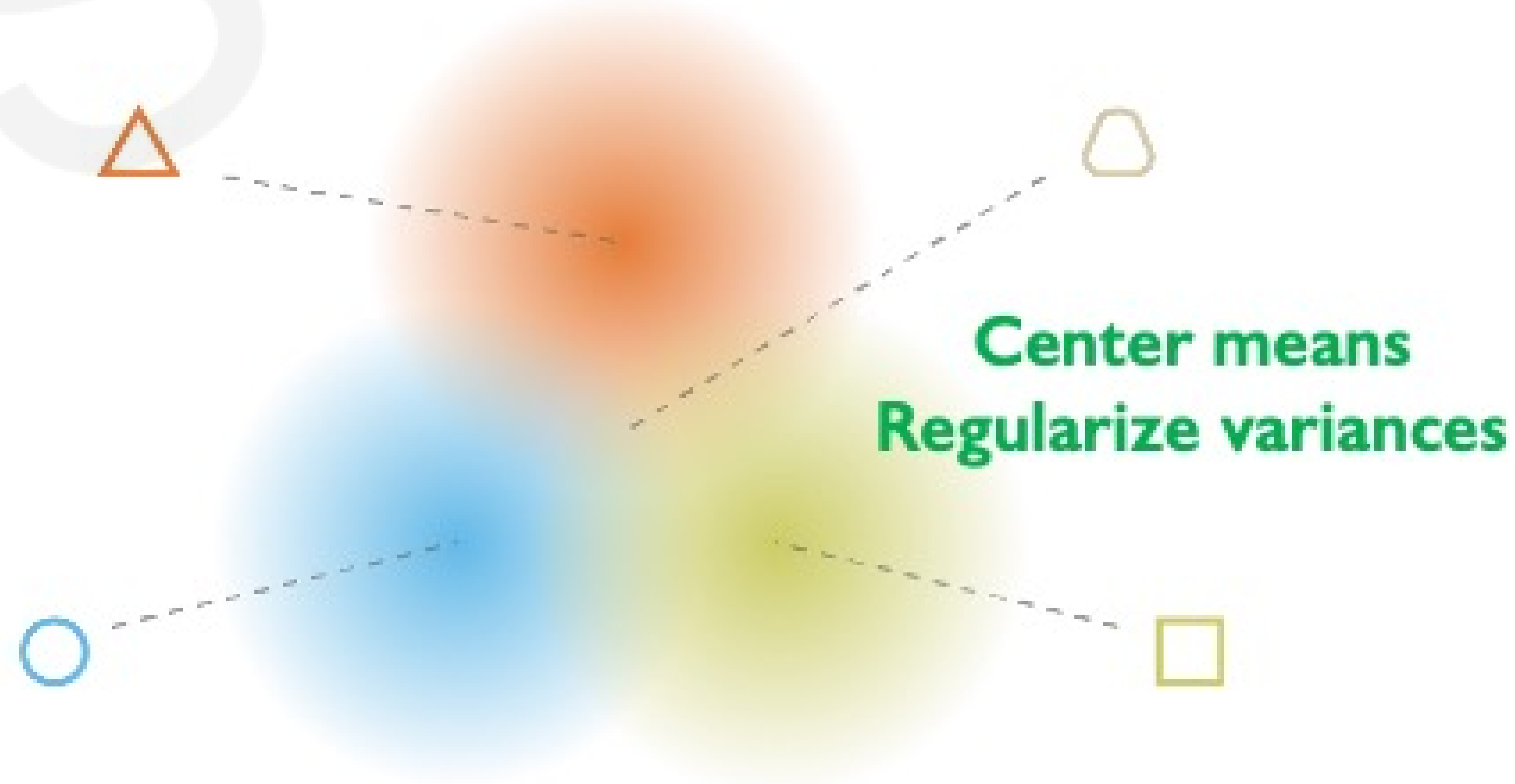
Intuition on regularization and the Normal prior

1. **Continuity**: points that are close in latent space → similar content after decoding
2. **Completeness:** sampling from latent space → "meaningful" content after decoding

Encoding as a distribution does not guarantee these properties!

Not regularized

Normal prior →
continuity + completeness



Regularized

Intuition on regularization and the Normal prior

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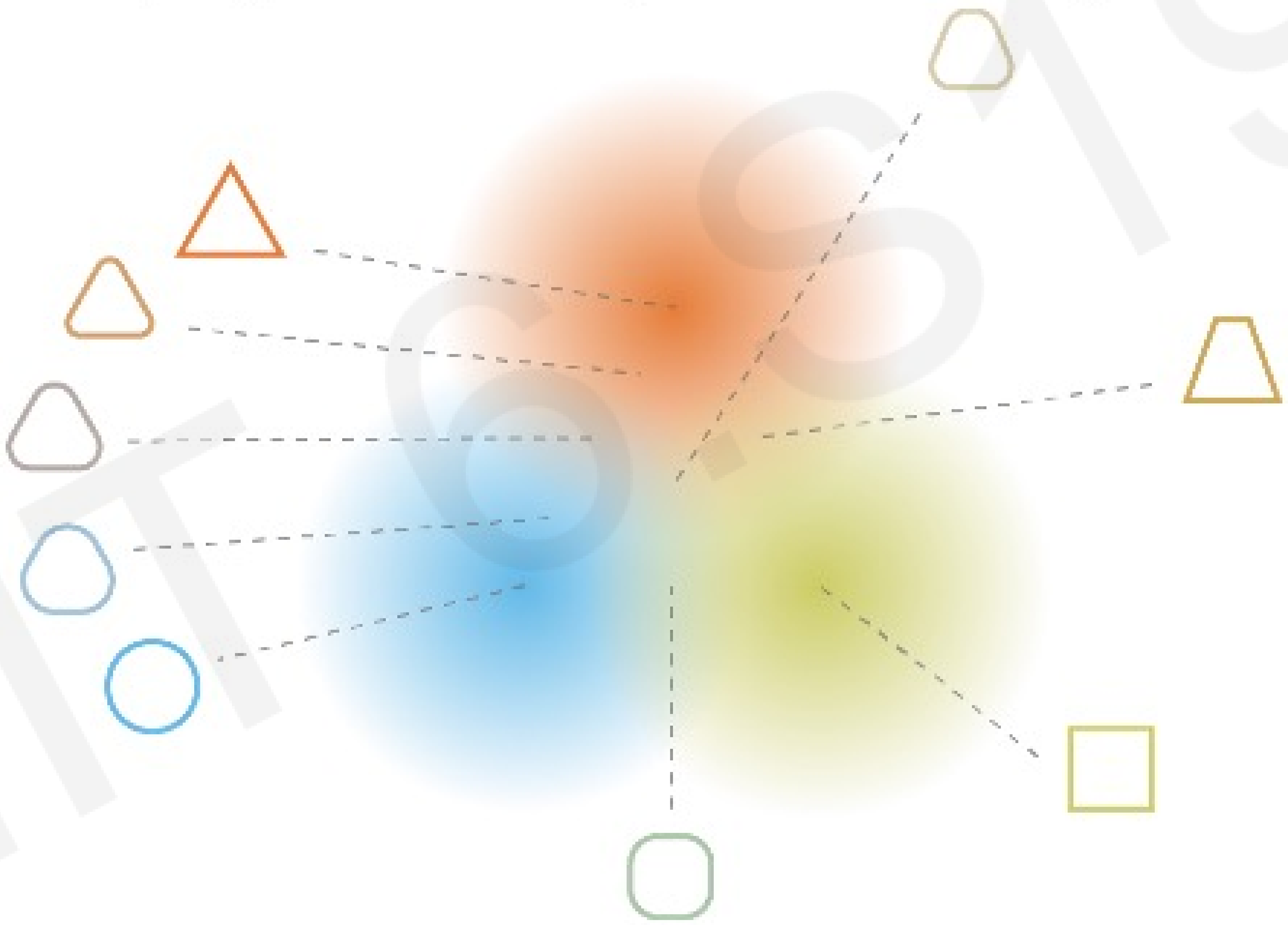


Regularization with Normal prior helps enforce **continuity & completeness** in the latent space.

VAE computation graph

$$\mathcal{L}(\phi, \theta, x) = (\text{reconstruction loss}) + (\text{regularization term})$$

VAE computation graph

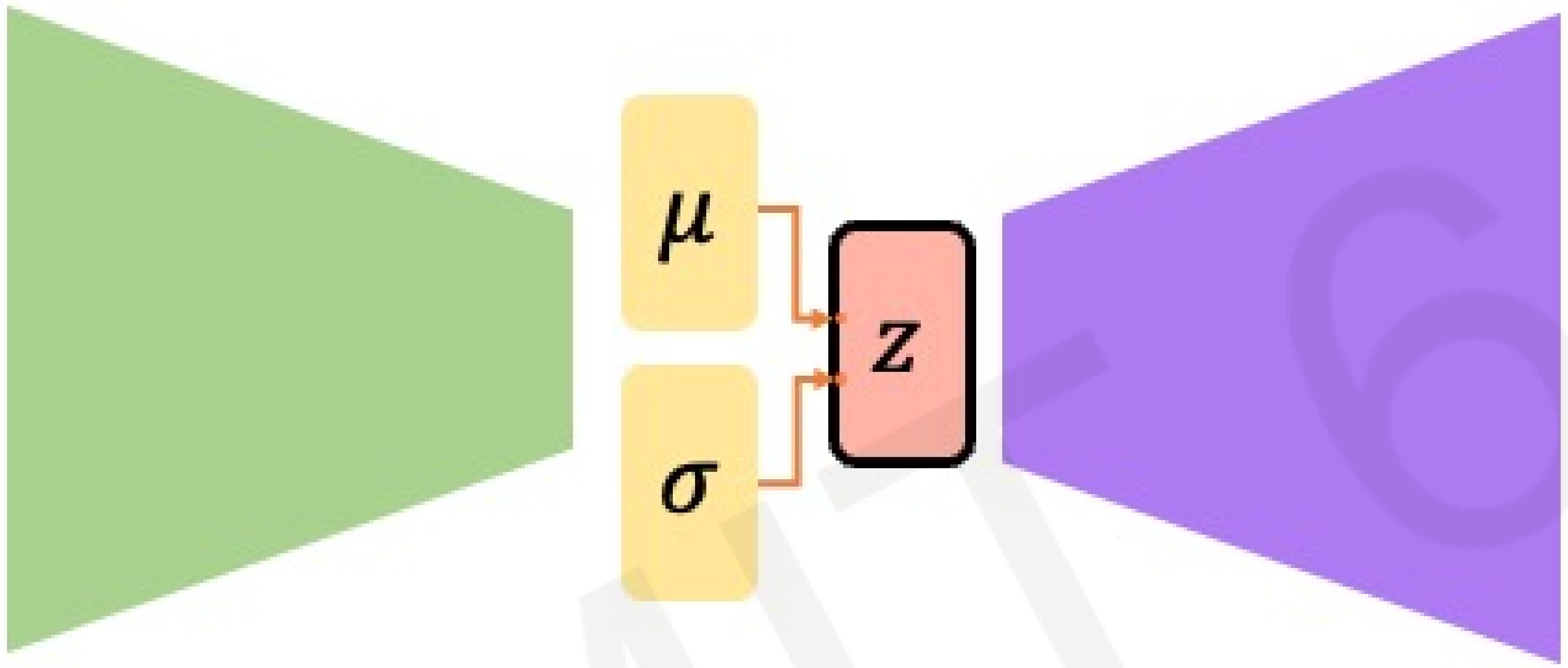
Problem: We cannot backpropagate gradients through sampling layers!

Encoder computes: $q_\phi(z|x)$

Decoder computes: $p_\theta(x|z)$

$$\mathcal{L}(\phi, \theta, x) = (\text{reconstruction loss}) + (\text{regularization term})$$

Reparametrizing the sampling layer



Key Idea:

$$\cancel{z \sim \mathcal{N}(\mu, \sigma^2)}$$

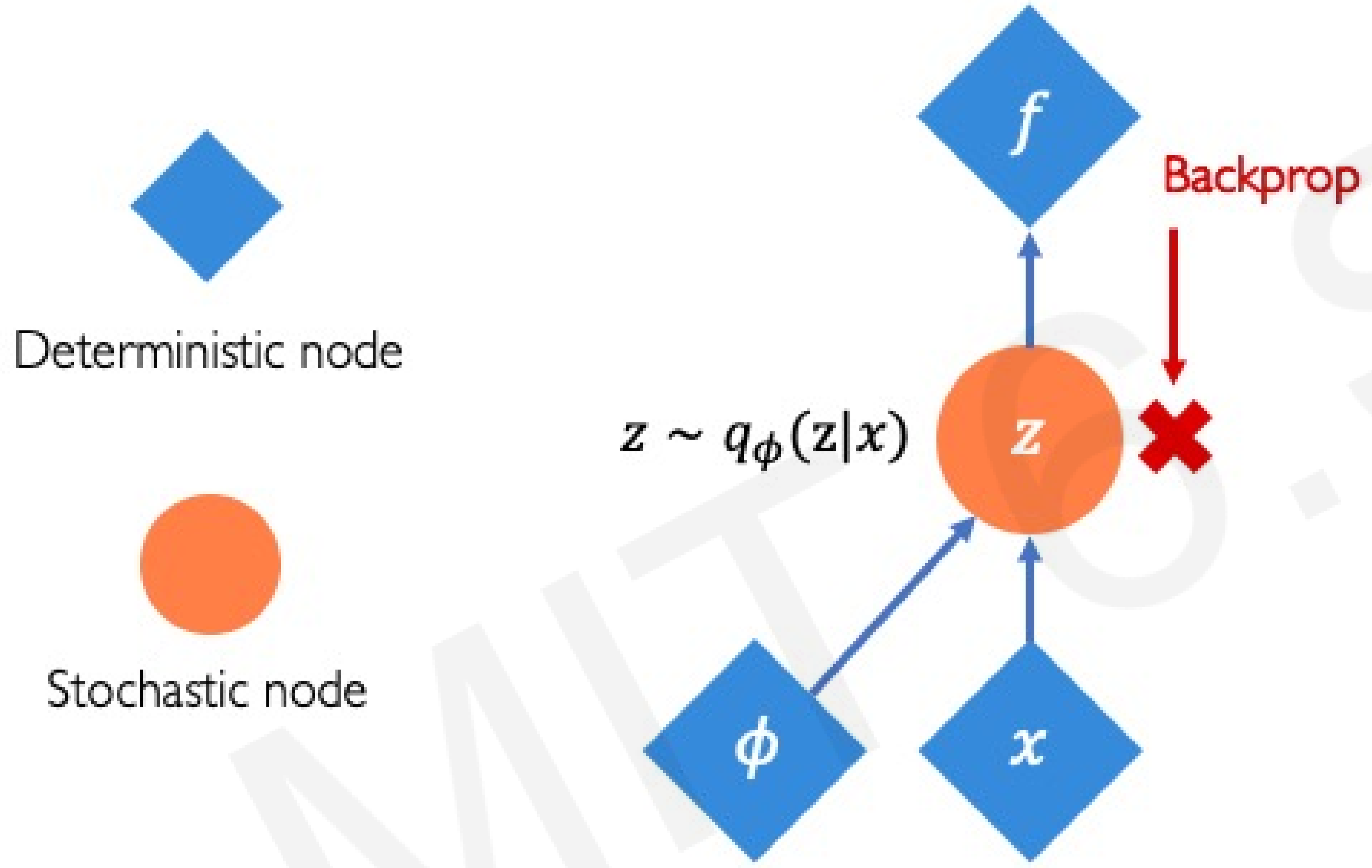
Consider the sampled latent vector z as a sum of

- a fixed μ vector,
- and fixed σ vector, scaled by random constants drawn from the prior distribution

$$\Rightarrow z = \mu + \sigma \odot \varepsilon$$

where $\varepsilon \sim \mathcal{N}(0,1)$

Reparametrizing the sampling layer



Original form

Reparametrizing the sampling layer

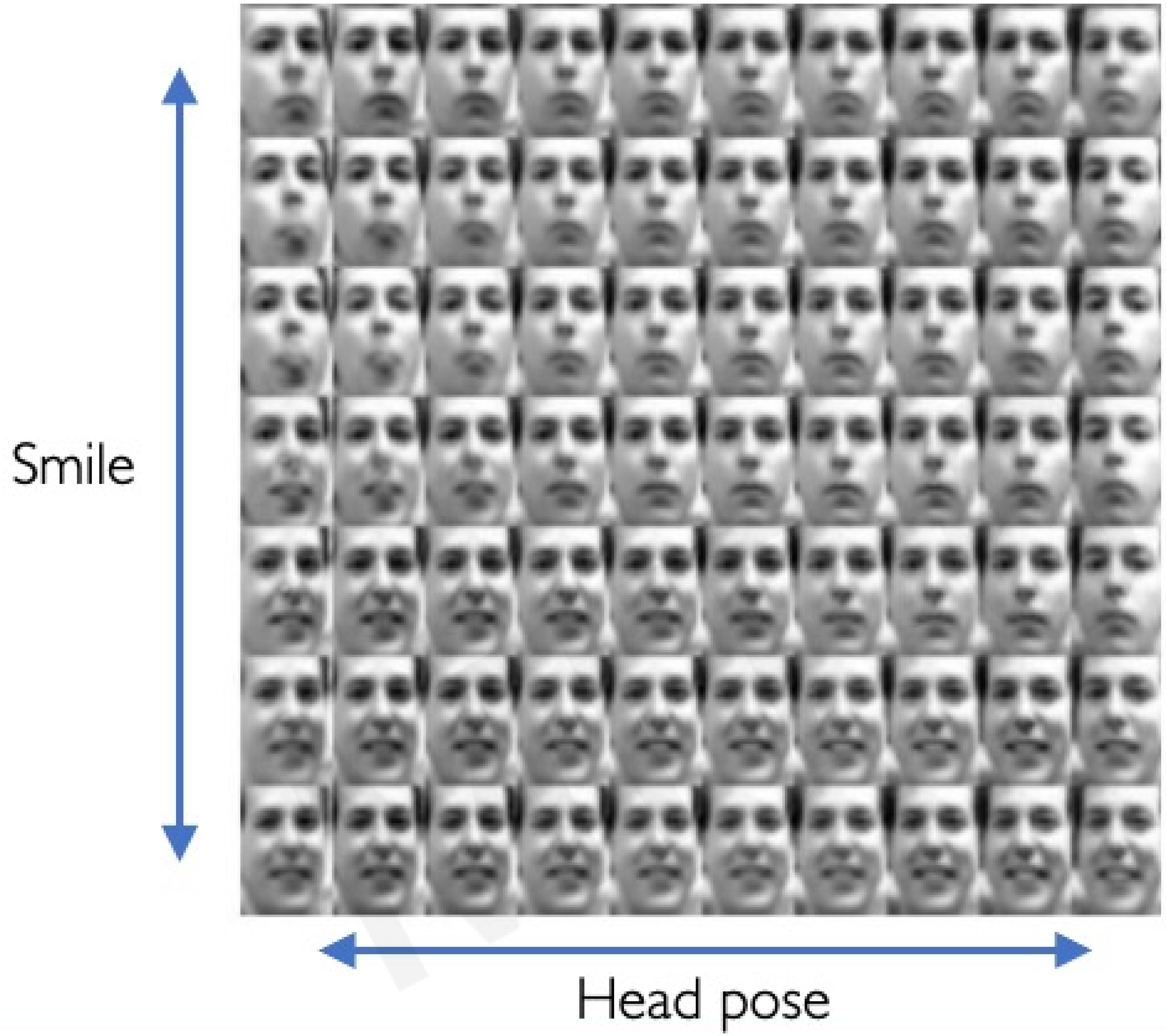
VAEs: Latent perturbation

Slowly increase or decrease a **single latent variable**
Keep all other variables fixed

Head pose

Different dimensions of z encodes **different interpretable latent features**

VAEs: Latent perturbation



Ideally, we want latent variables that are uncorrelated with each other

Enforce diagonal prior on the latent variables to encourage independence

Disentanglement

Latent space disentanglement with β-VAEs

β-VAE loss:

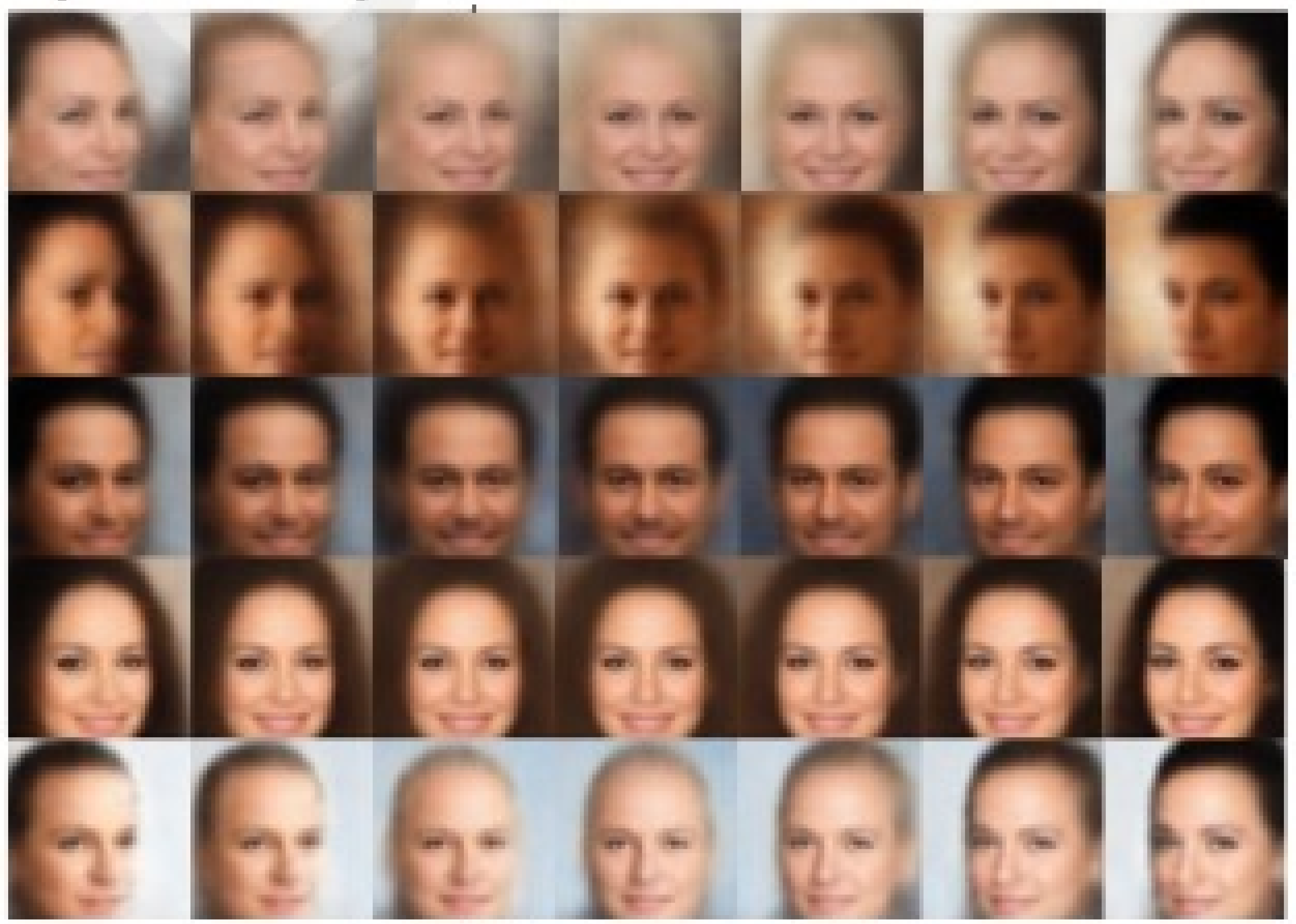
$$\mathcal{L}(\theta,\phi;\mathbf{x},\mathbf{z},\beta) = \underbrace{\mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})}[\log p_\theta(\mathbf{x}|\mathbf{z})]}_{\textbf{Reconstruction term}} - \boxed{}\underbrace{D_{KL}(q_\phi(\mathbf{z}|\mathbf{x}) \parallel p(\mathbf{z}))}_{\textbf{Regularization term}}$$

β > 1: constrain latent bottleneck, encourage efficient latent encoding → disentanglement

Head rotation (azimuth)

Smile also changing!

Standard VAE (β = 1)



β-VAE (β = 250)

Smile relatively constant!

Why latent variable models? Debiasing

Capable of uncovering **underlying latent variables** in a dataset

Homogeneous skin color, pose

vs

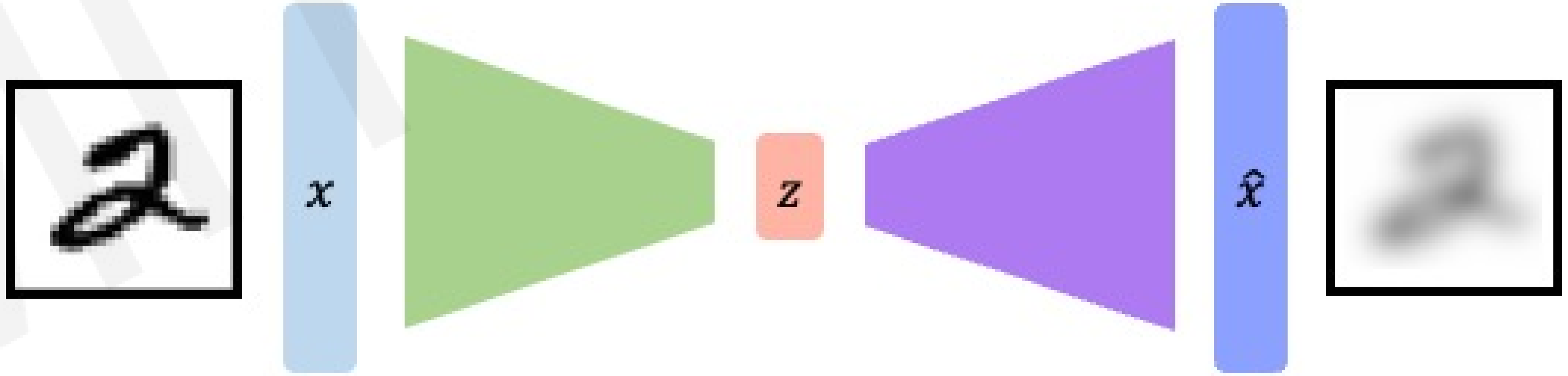
Diverse skin color, pose, illumination

How can we use latent distributions to create fair and representative datasets?

Software Lab!

VAE summary

1. Compress representation of world to something we can use to learn



VAE summary

1. Compress representation of world to something we can use to learn
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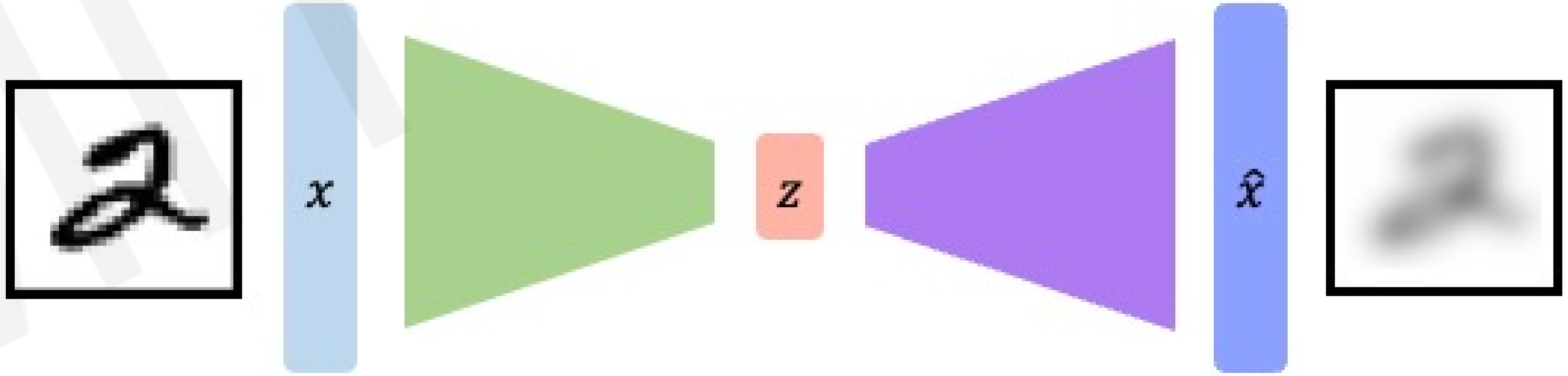

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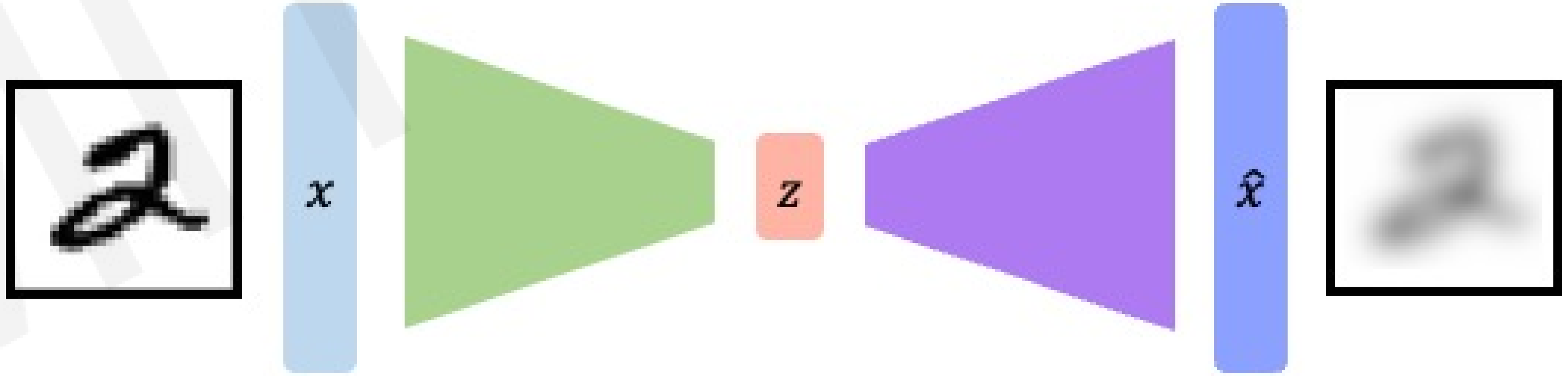

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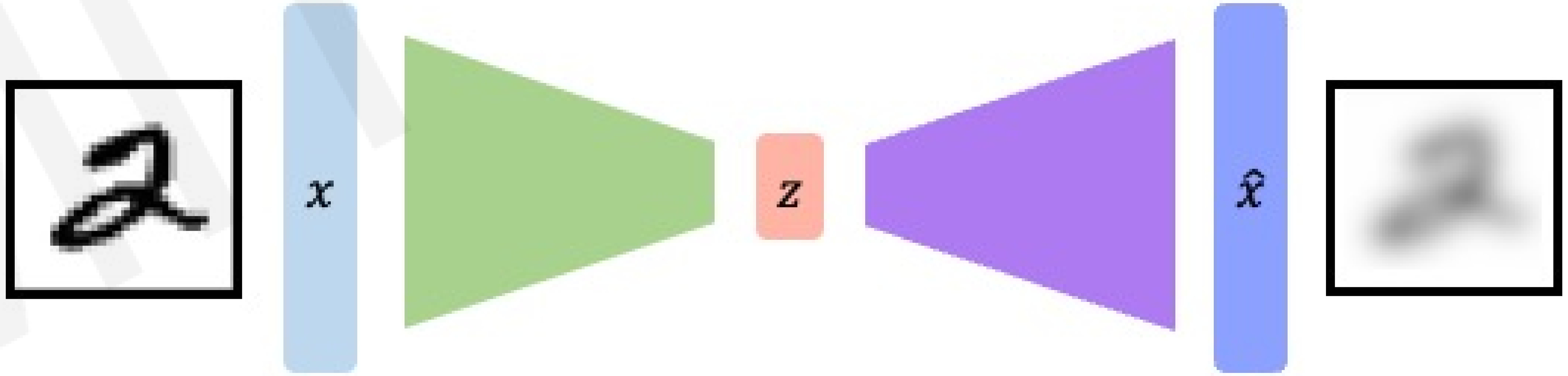
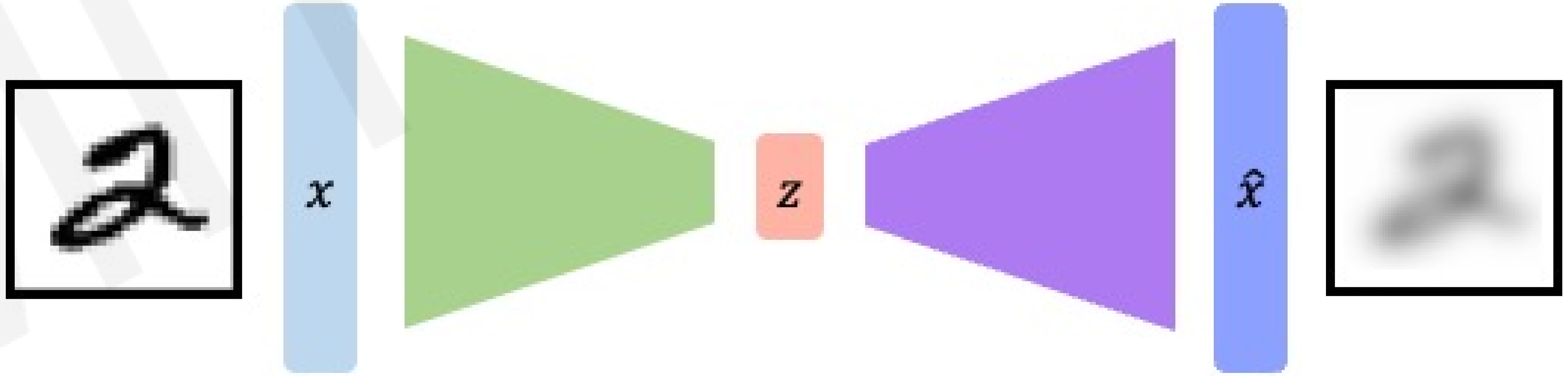


VAE summary

1. Compress representation of world to something we can use to learn
2. Reconstruction allows for unsupervised learning (no labels!)
3. Reparameterization trick to train end-to-end
4. Interpret hidden latent variables using perturbation
5. Generating new examples



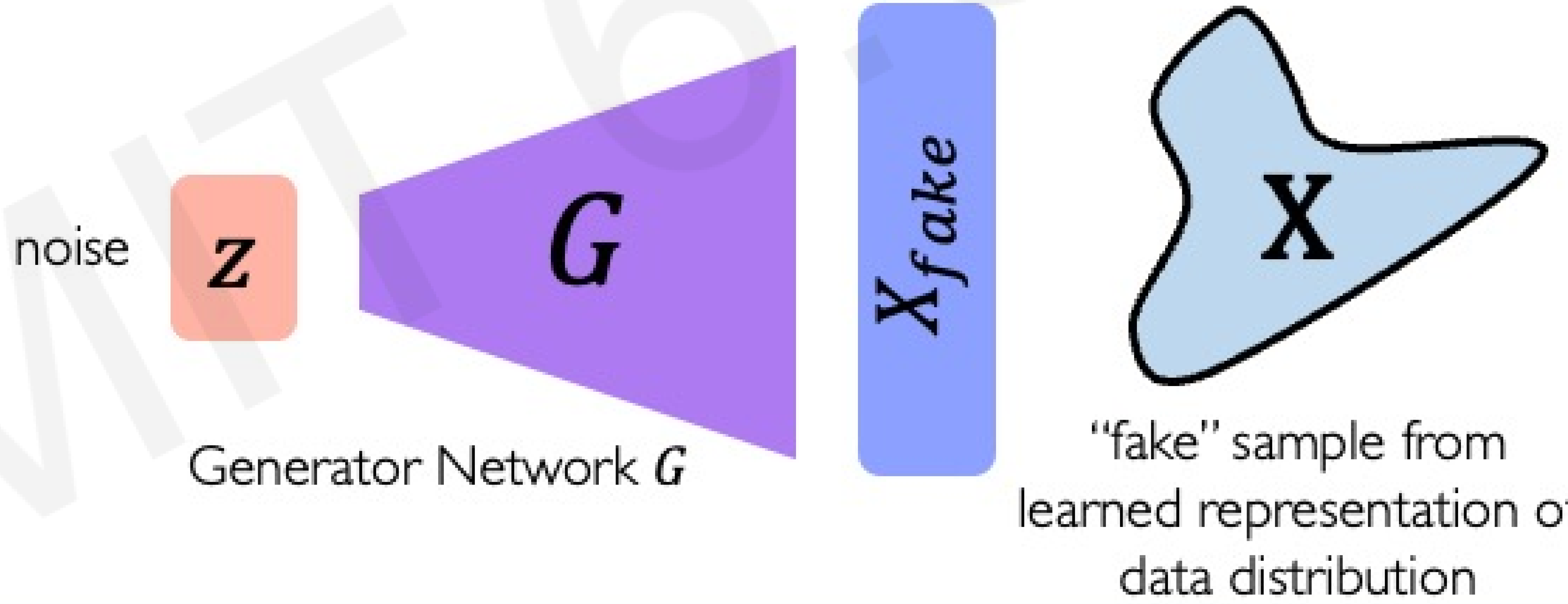
Generative Adversarial Networks (GANs)

What if we just want to sample?

Idea: don't explicitly model density, and instead just sample to generate new instances.

Problem: want to sample from complex distribution – can't do this directly!

Solution: sample from something simple (e.g., noise), learn a transformation to the data distribution.

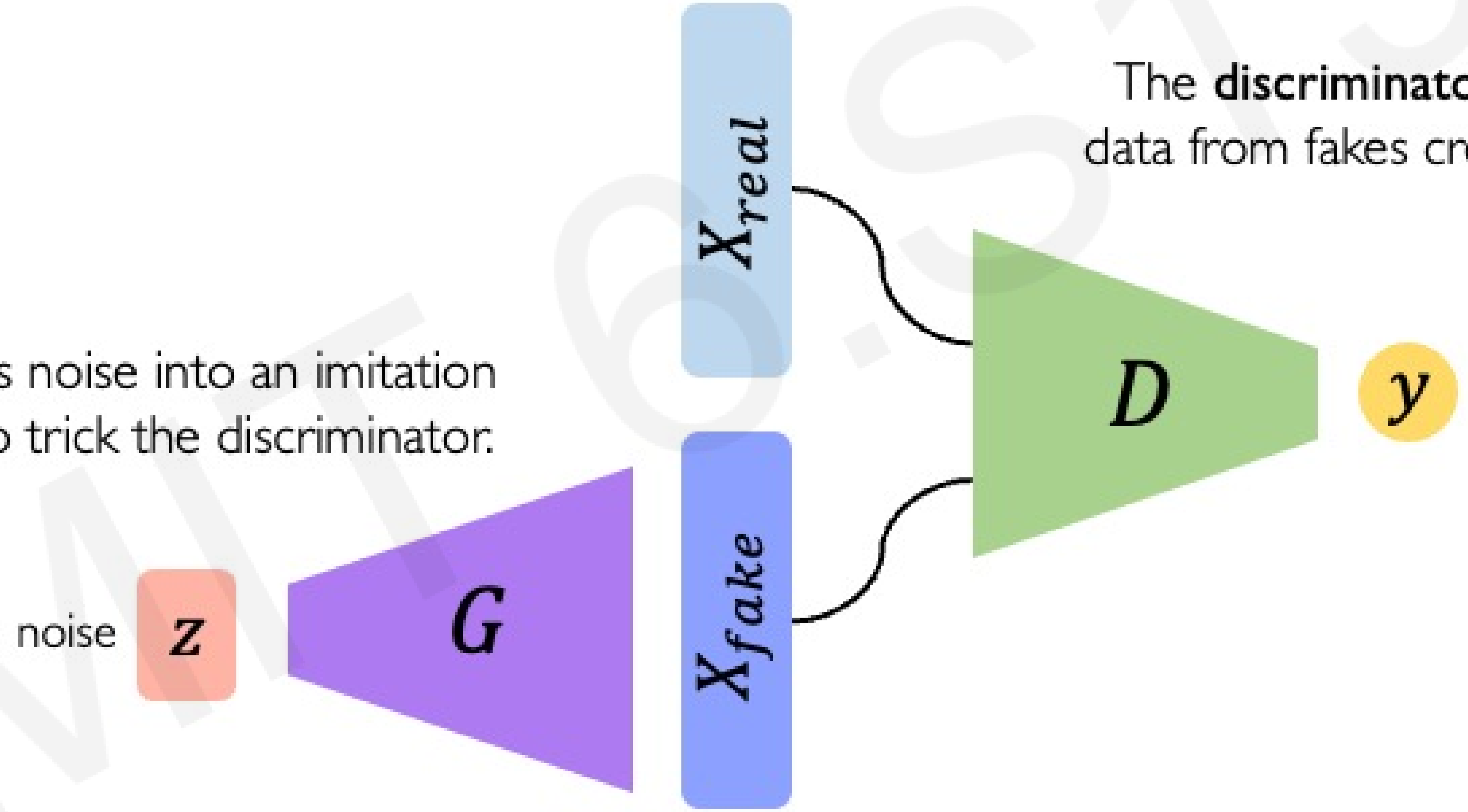


Generative Adversarial Networks (GANs)

Generative Adversarial Networks (GANs) are a way to make a generative model by having two neural networks compete with each other.

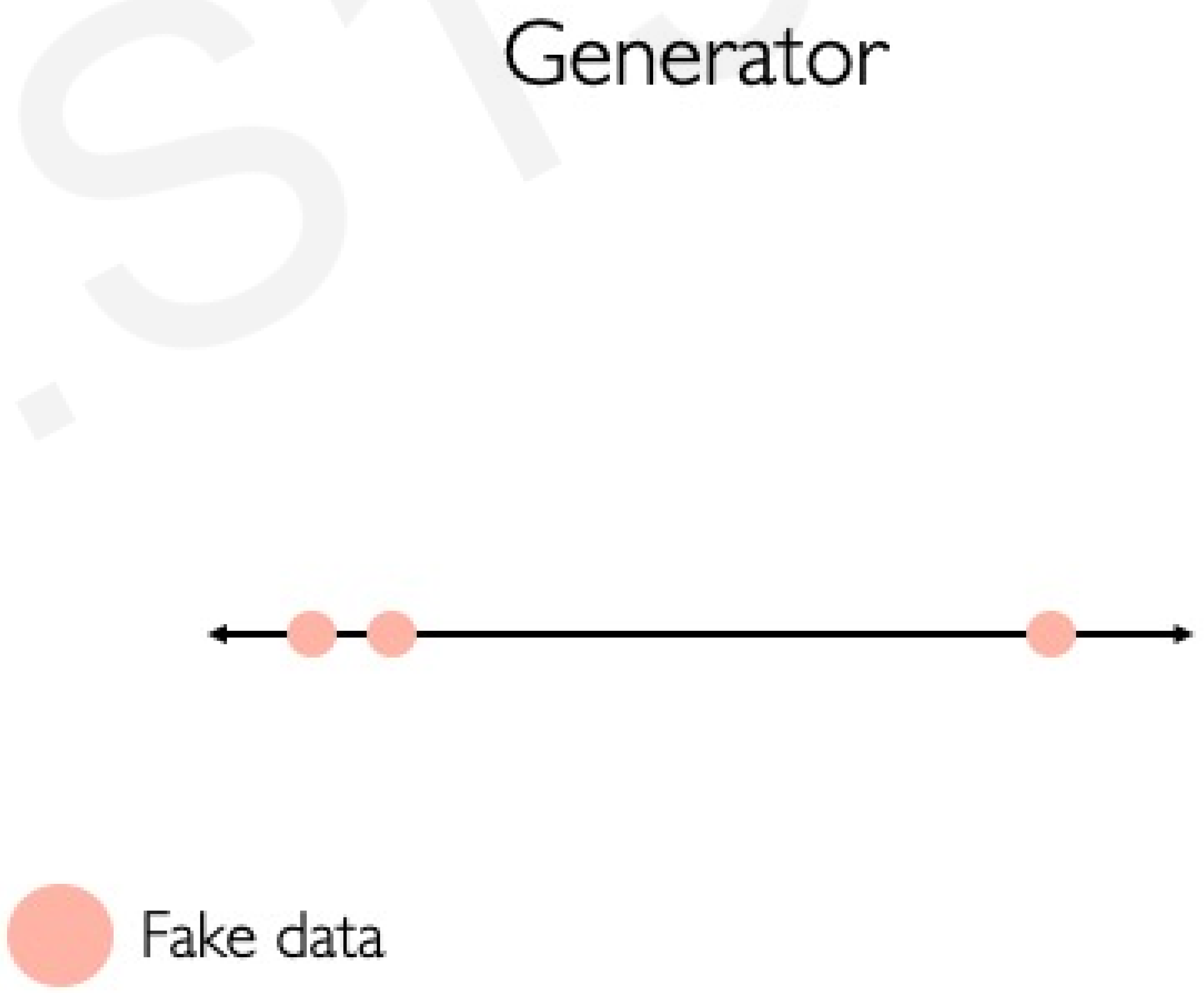
The **discriminator** tries to identify real data from fakes created by the generator.

The **generator** turns noise into an imitation of the data to try to trick the discriminator.



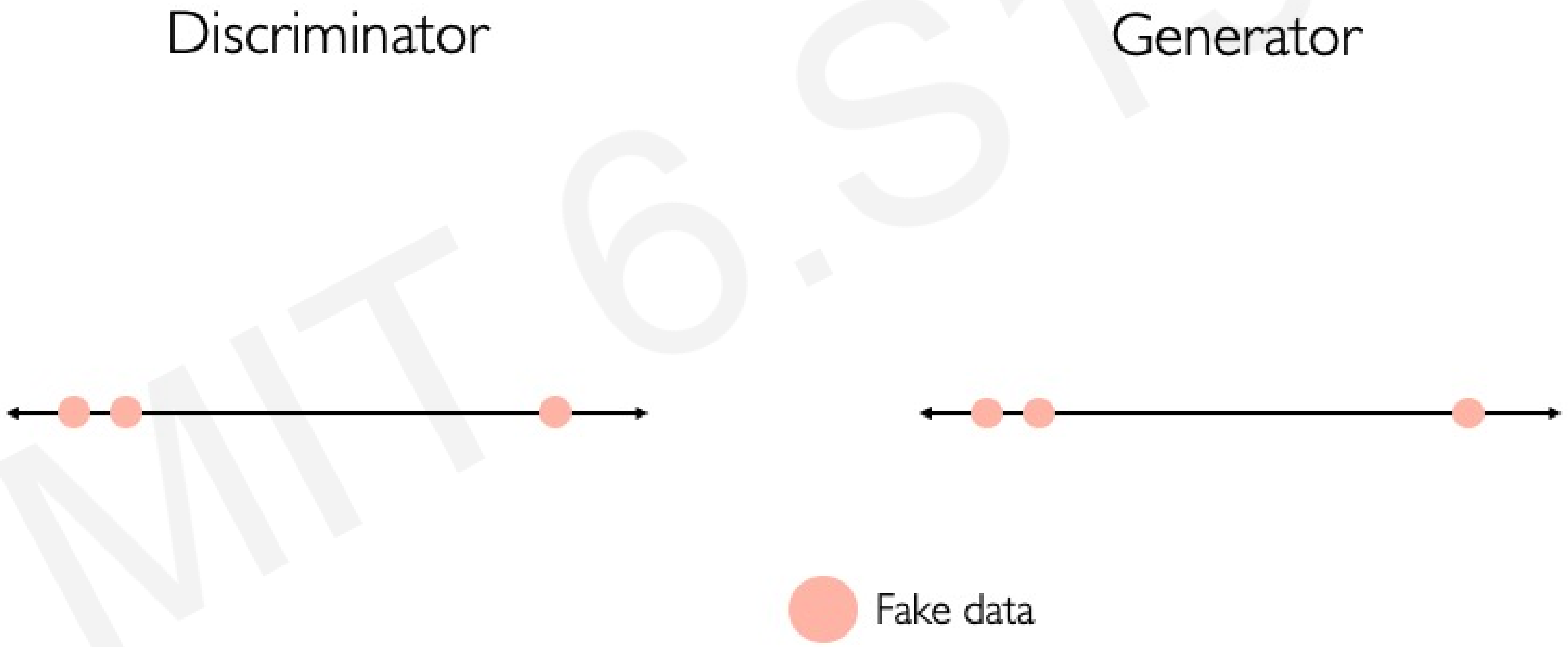
Intuition behind GANs

Generator starts from noise to try to create an imitation of the data.



Intuition behind GANs

Discriminator looks at both real data and fake data created by the generator.



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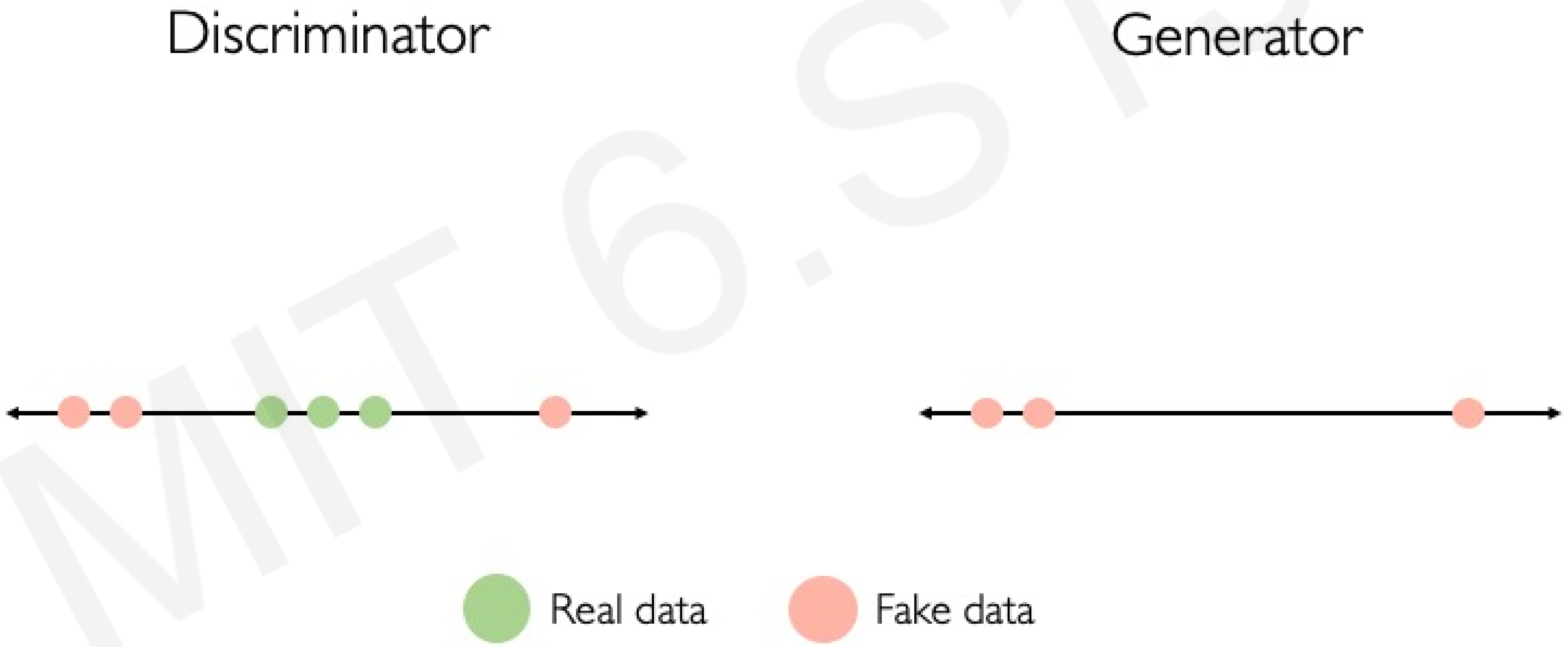
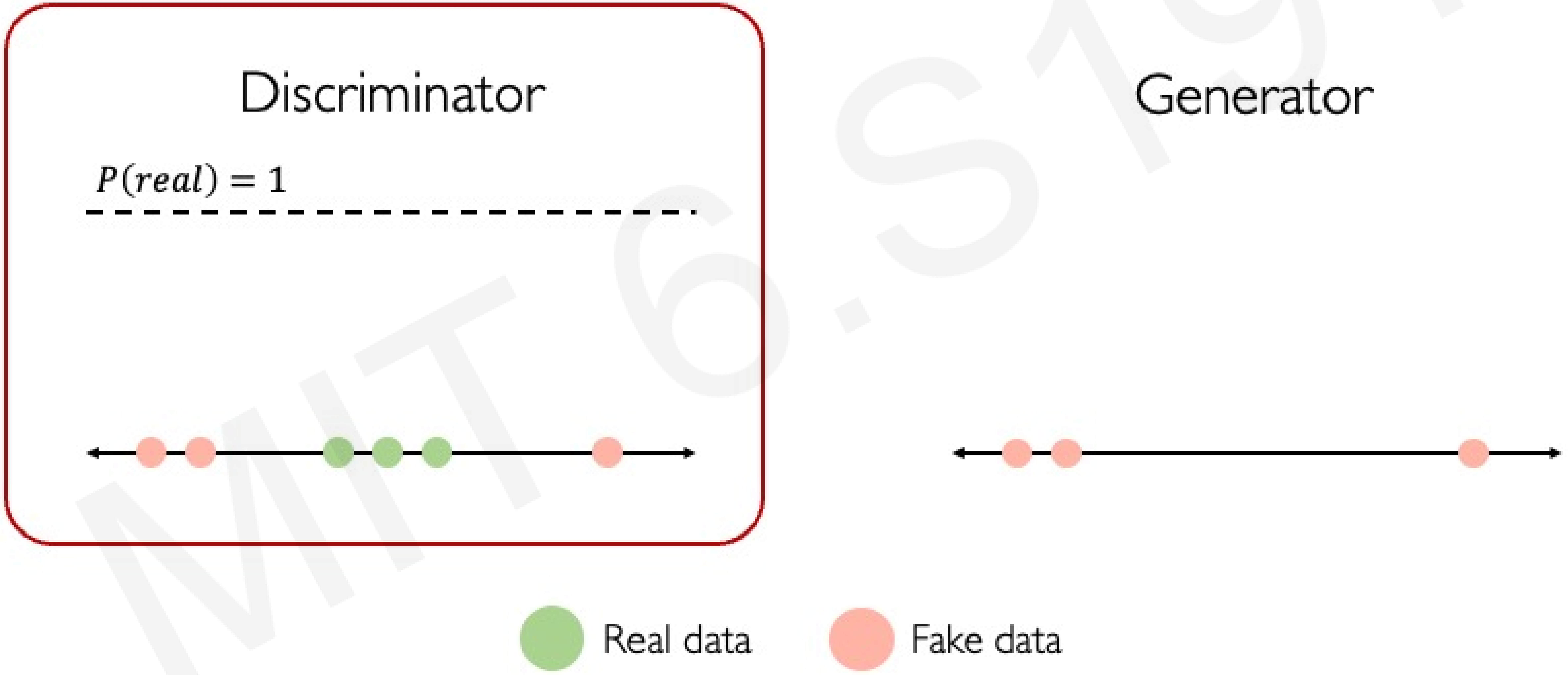


Intuition behind GANs

Discriminator tries to predict what's real and what's fake.



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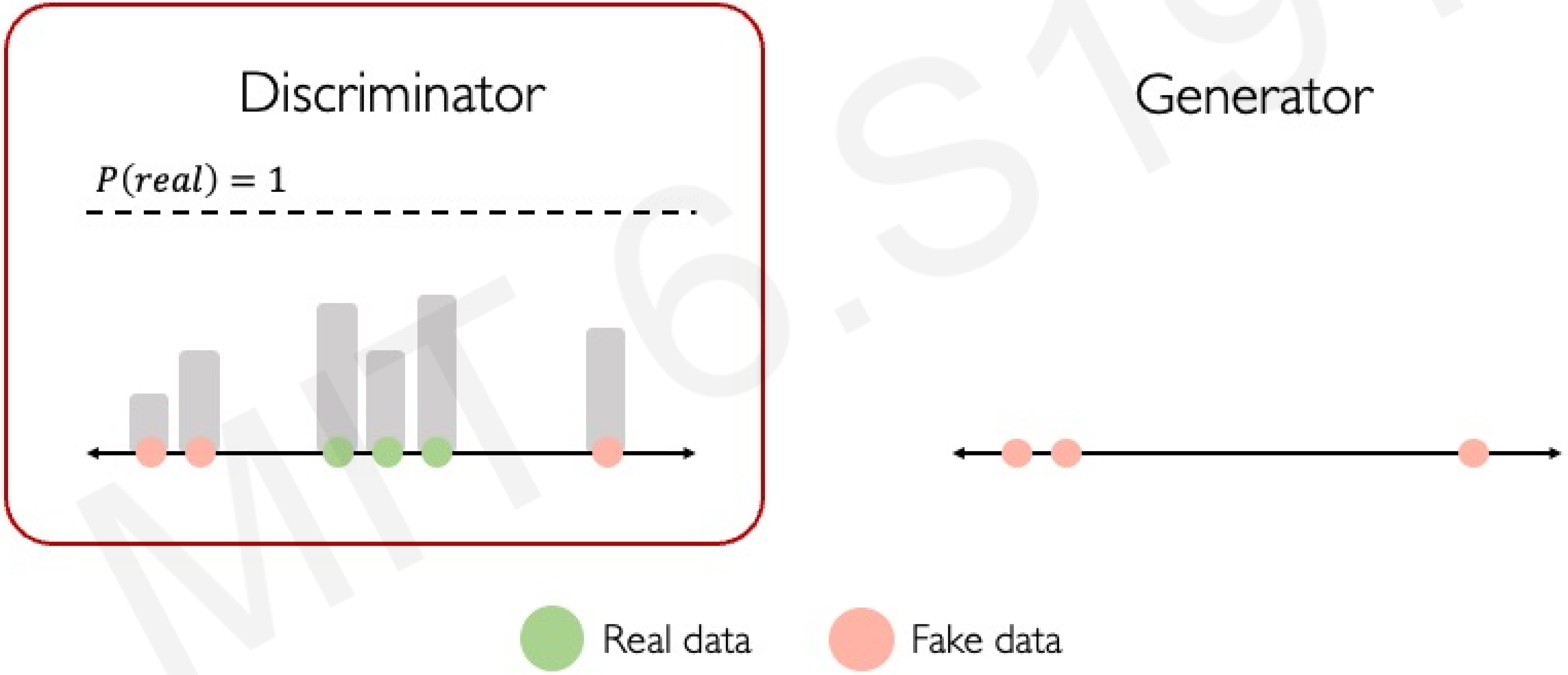

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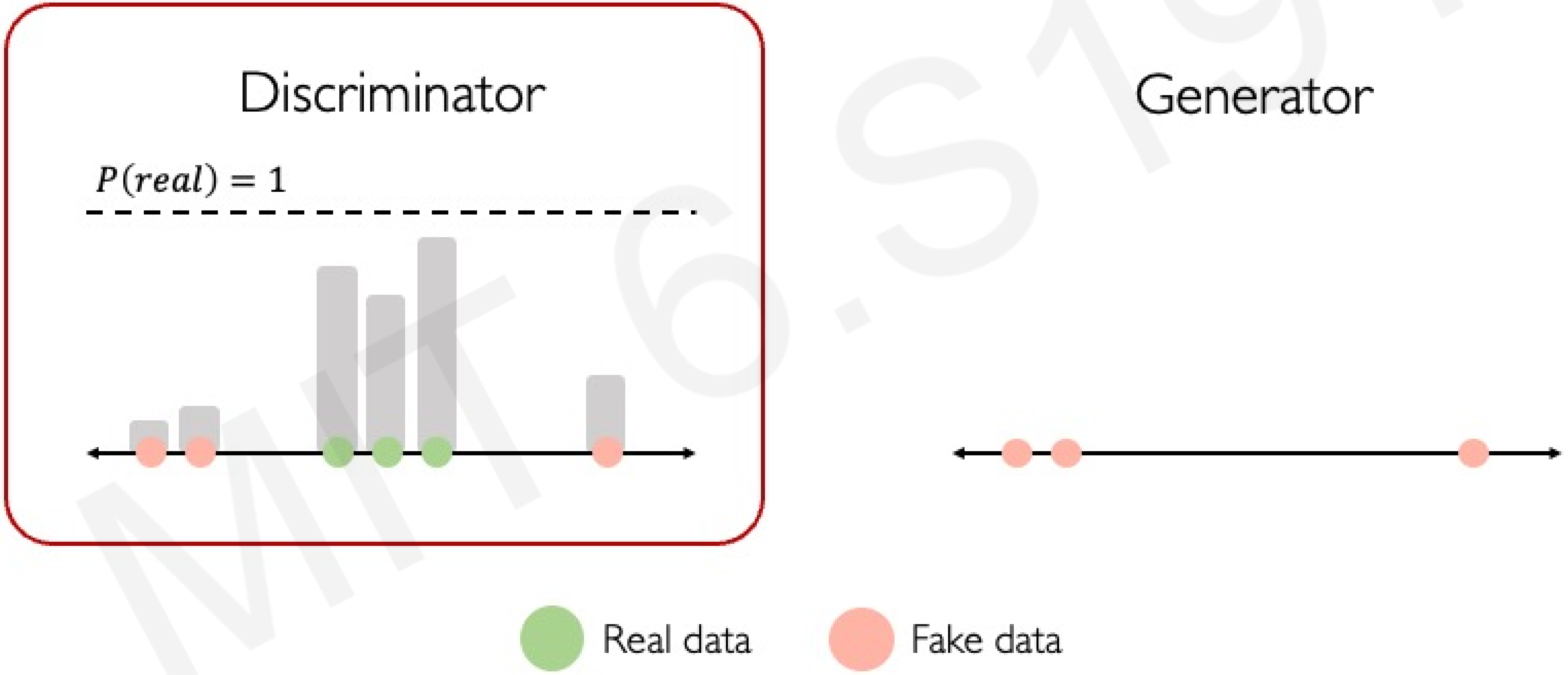

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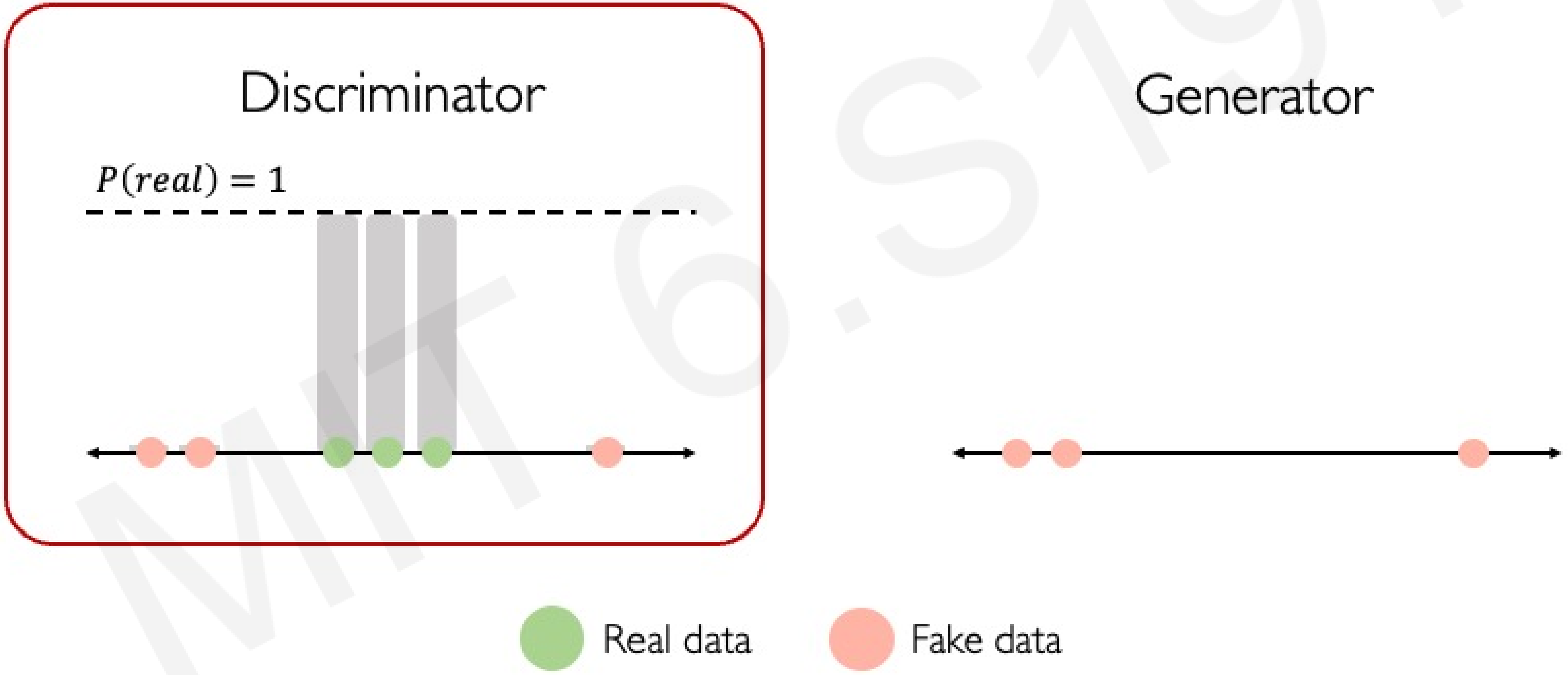
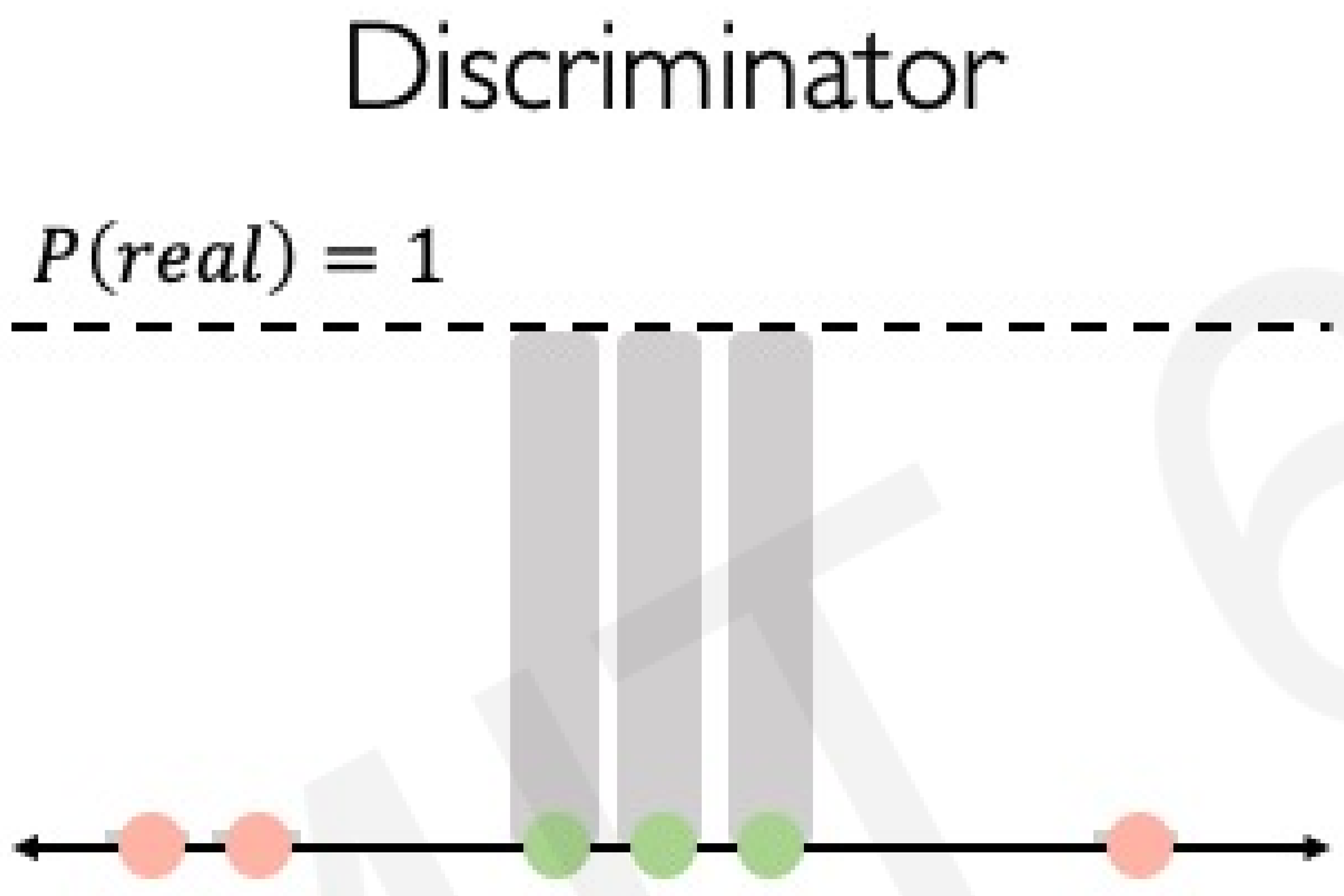
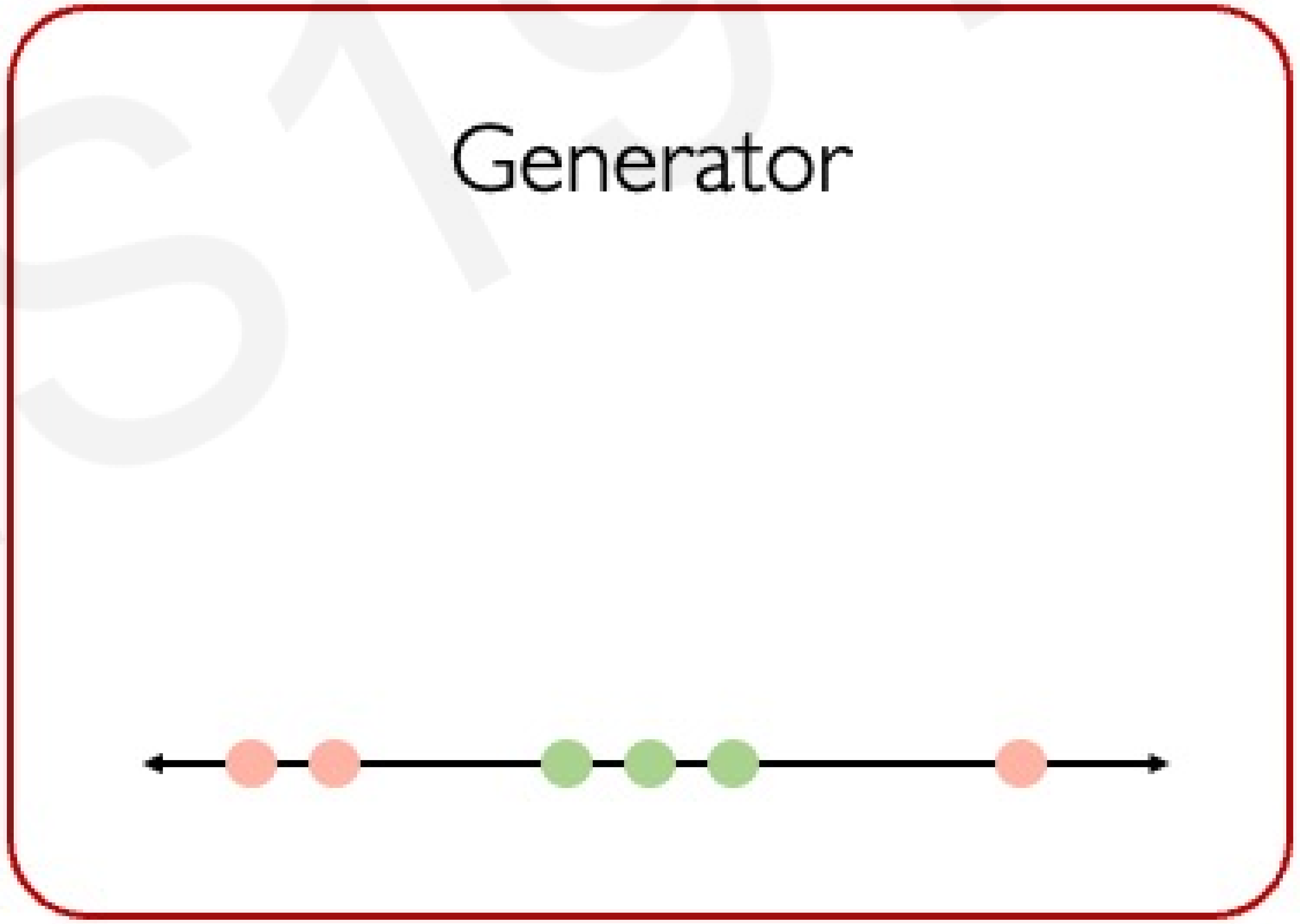


Intuition behind GANs

Generator tries to improve its imitation of the data.

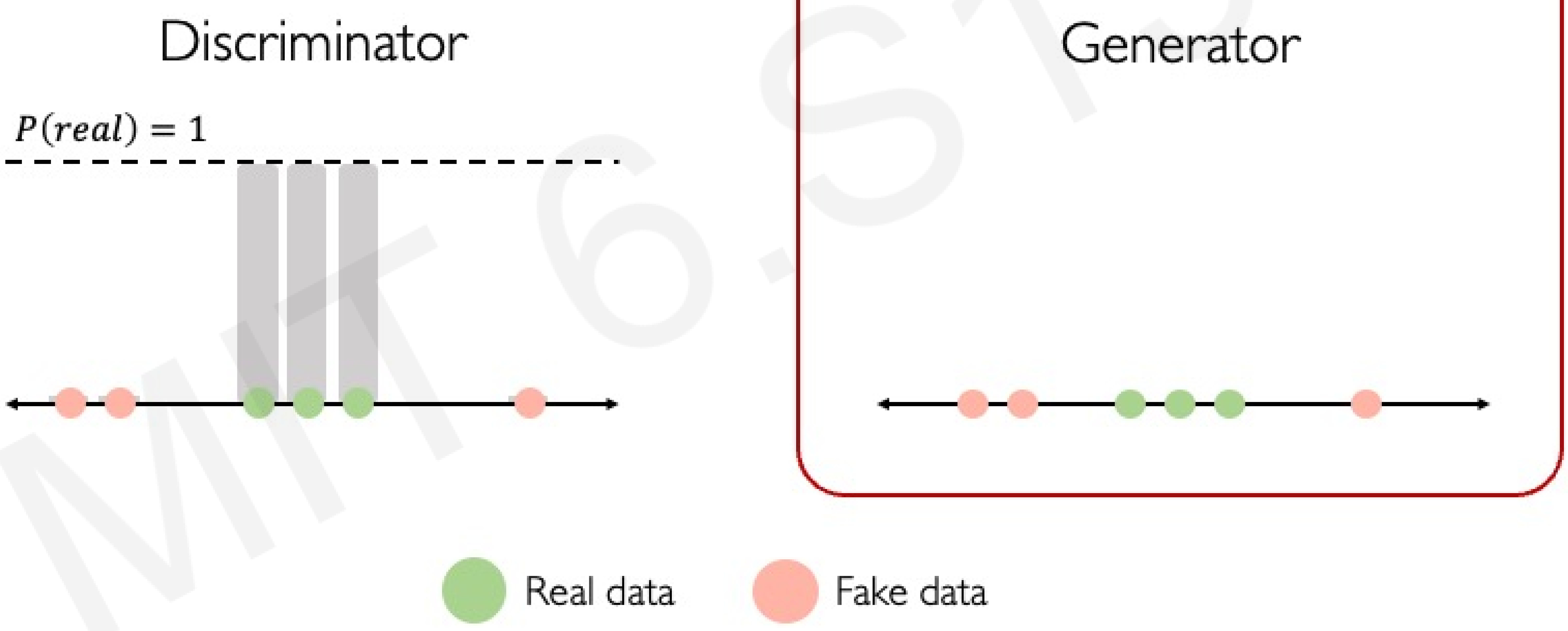




Real data Fake data

Intuition behind GANs

Generator tries to improve its imitation of the data.



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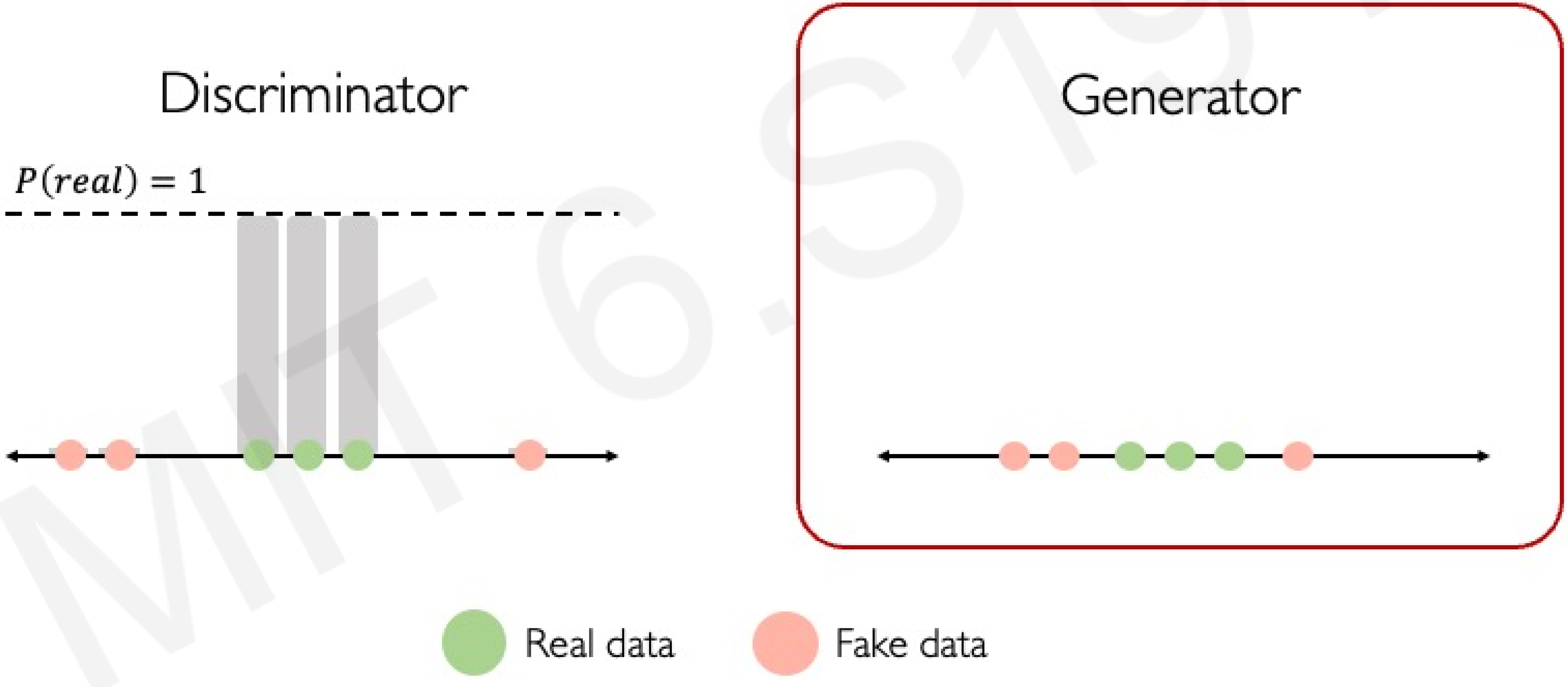

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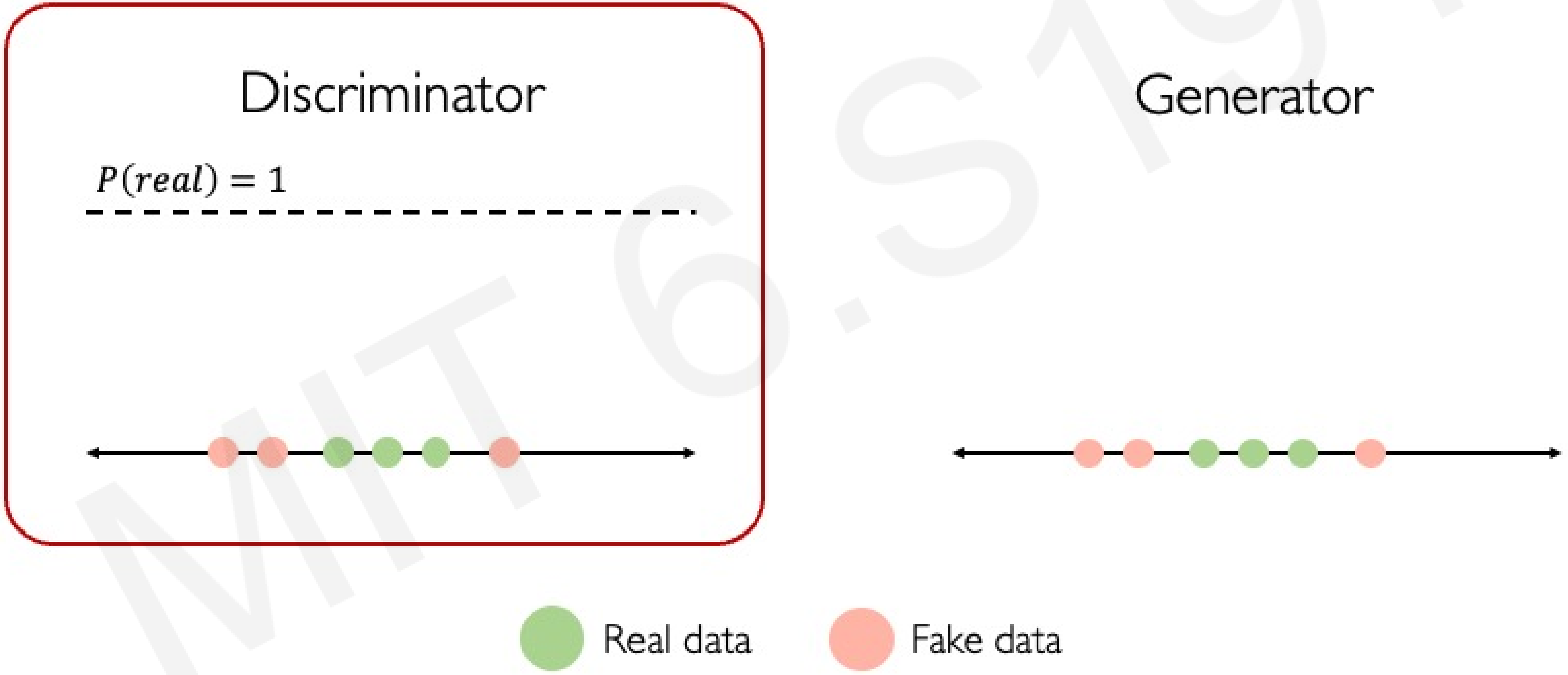

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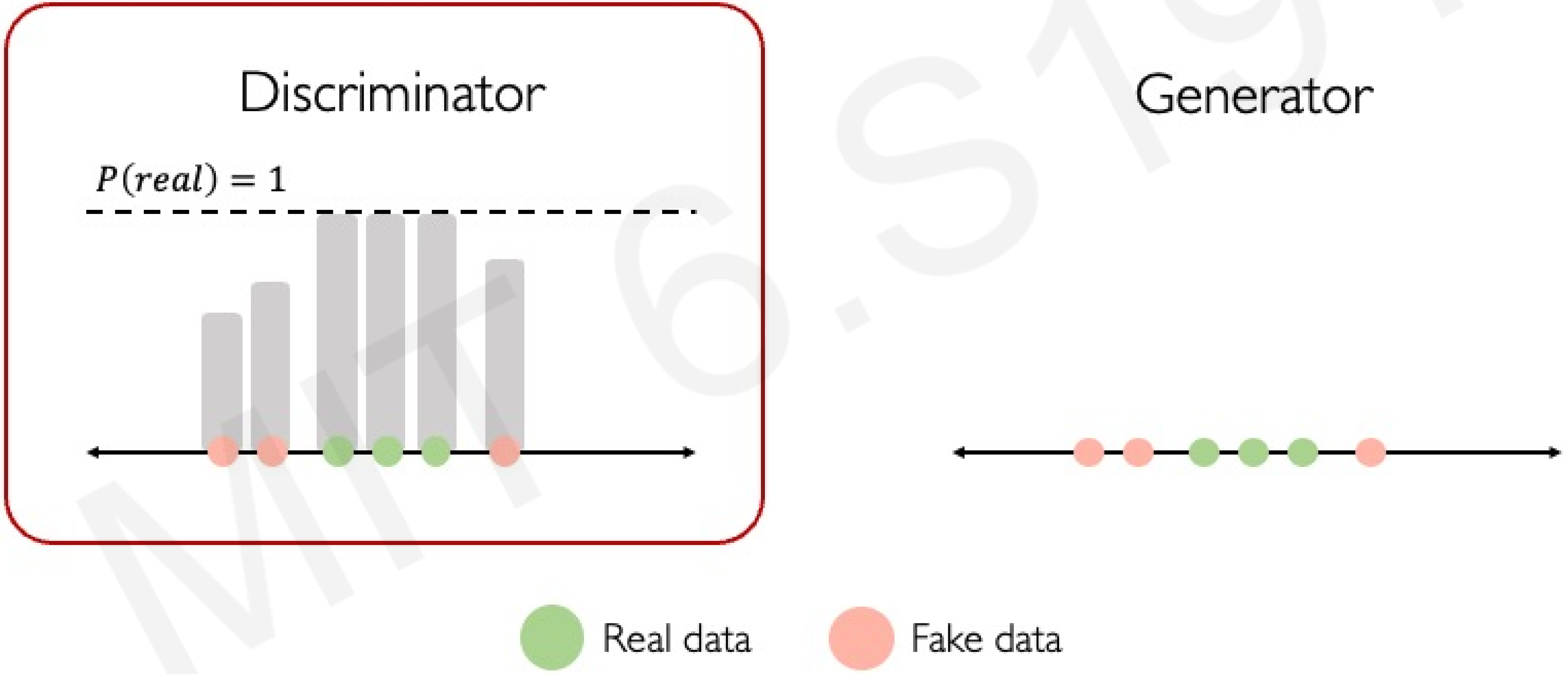

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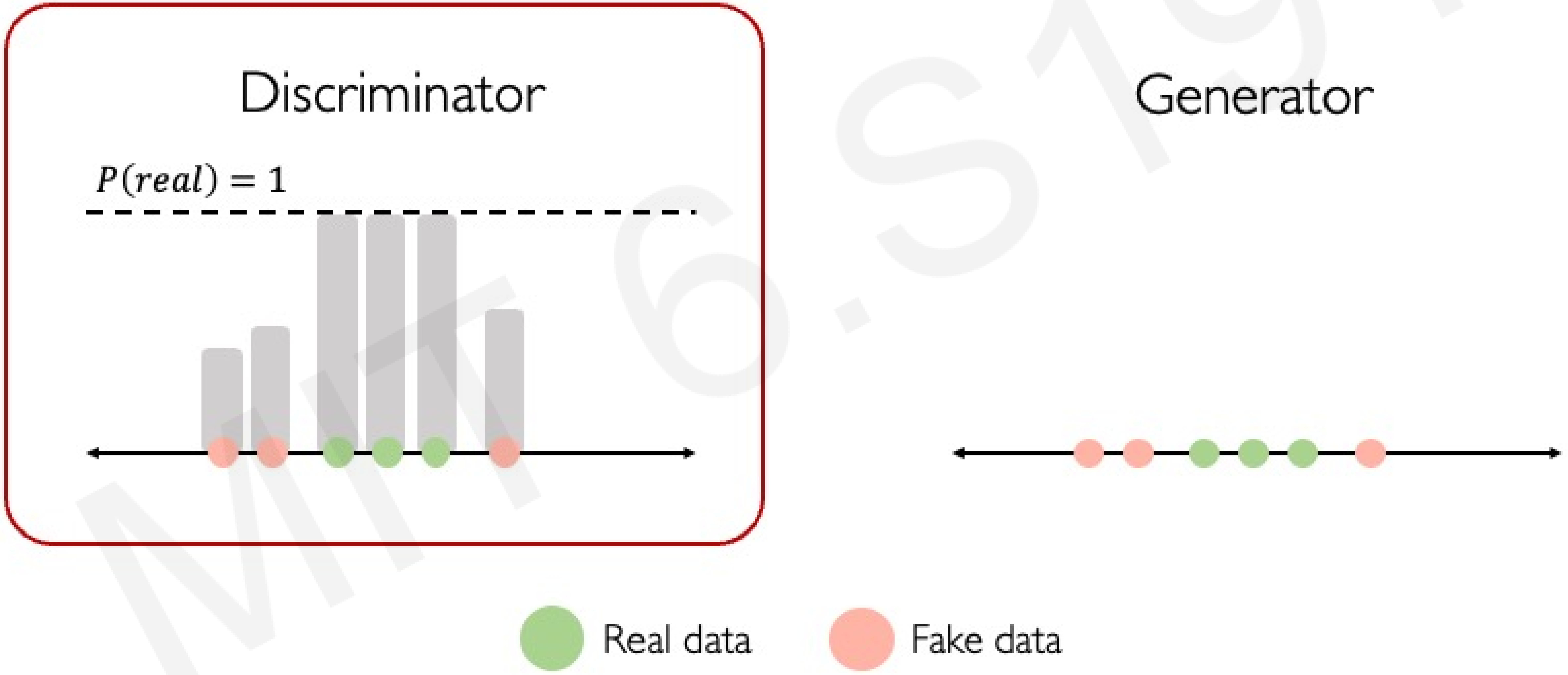

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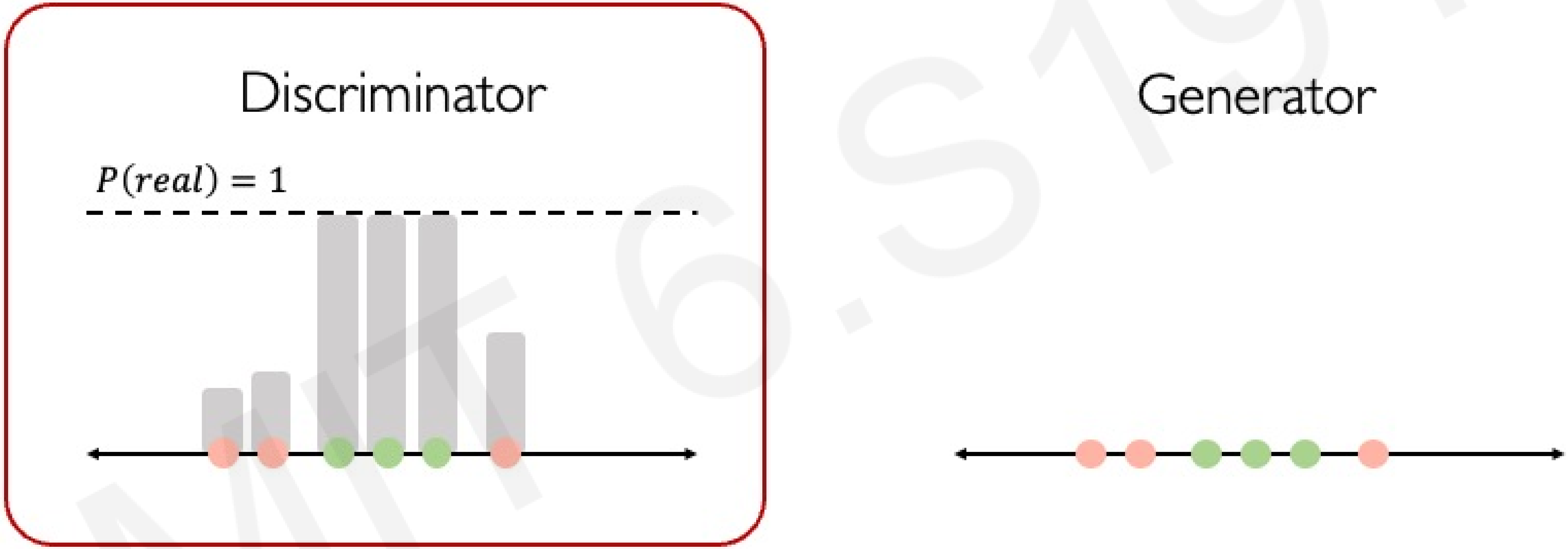

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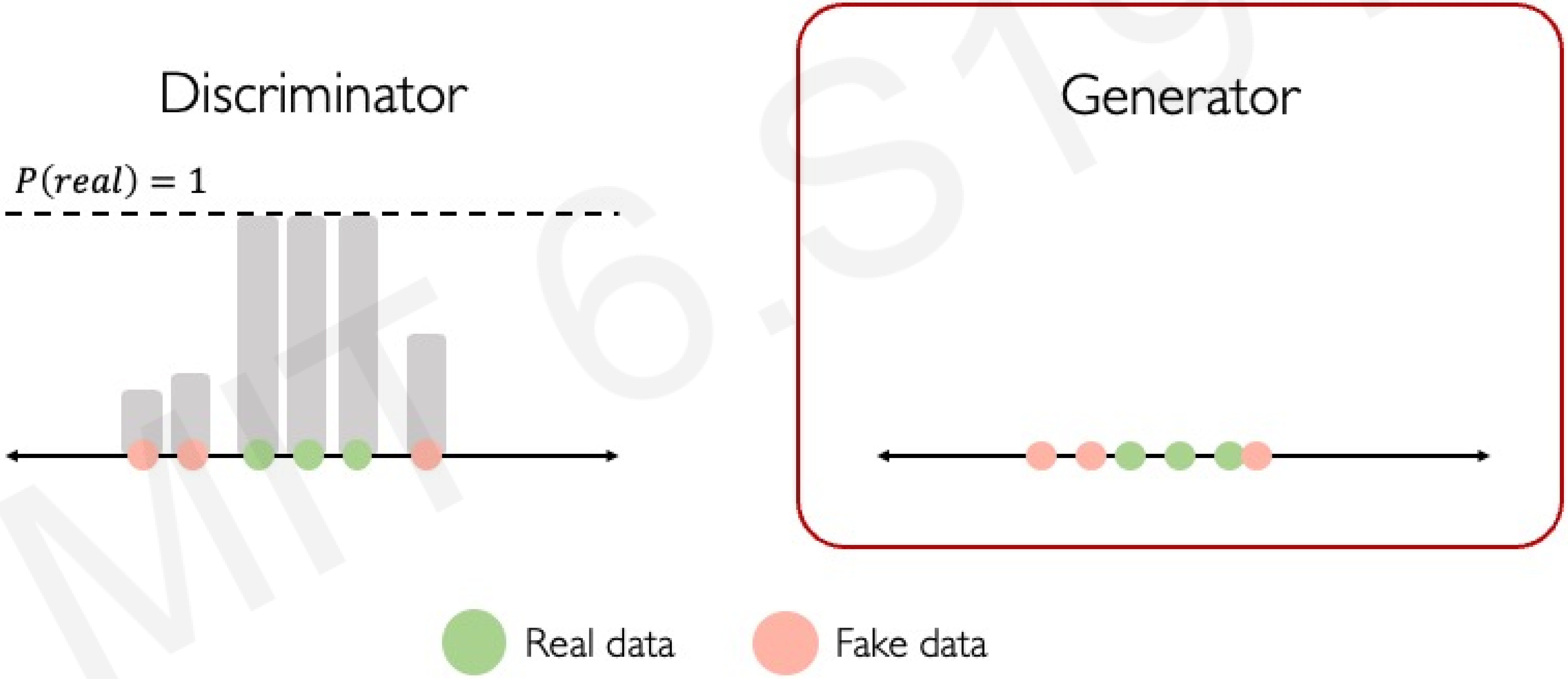

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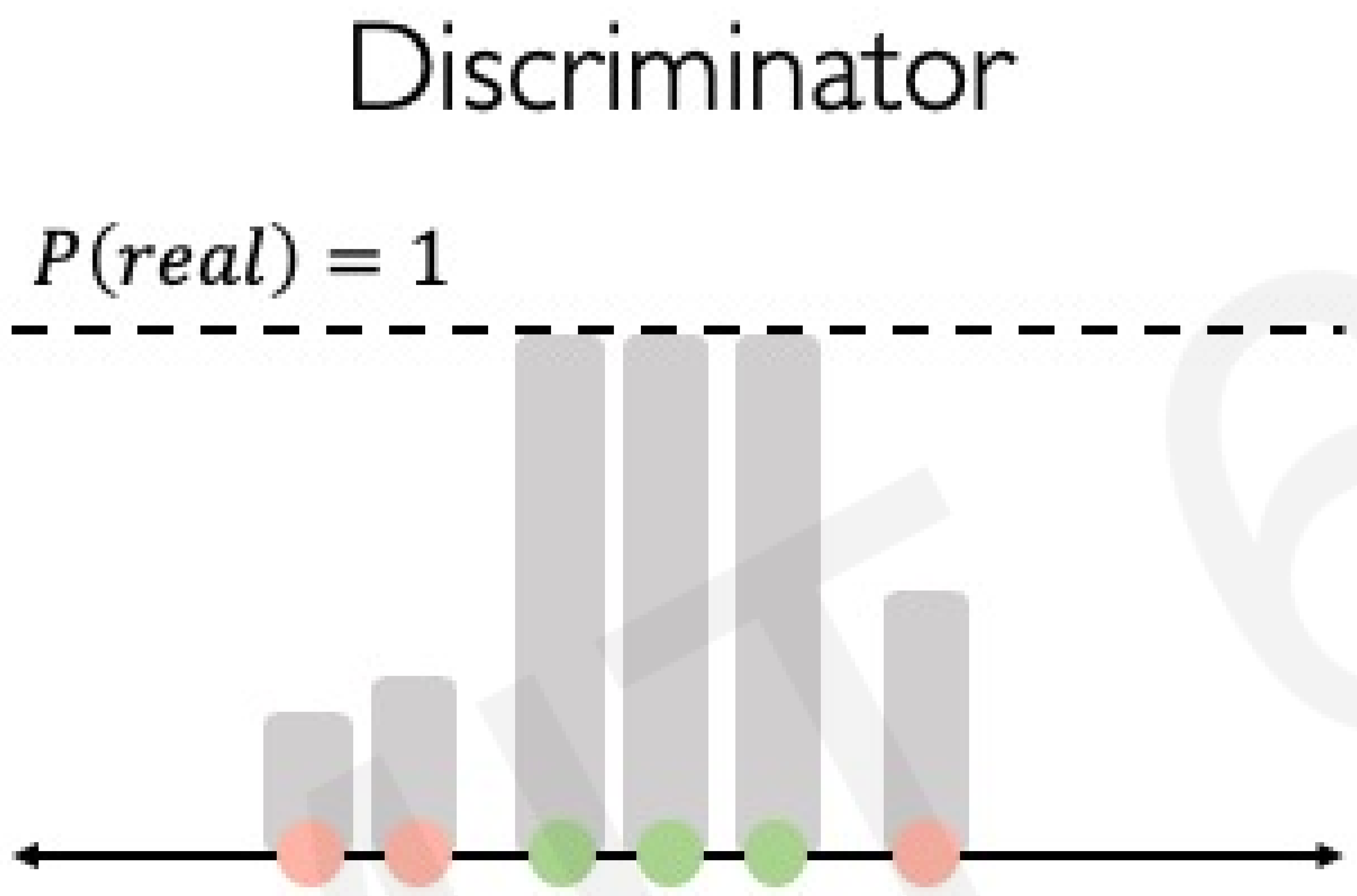

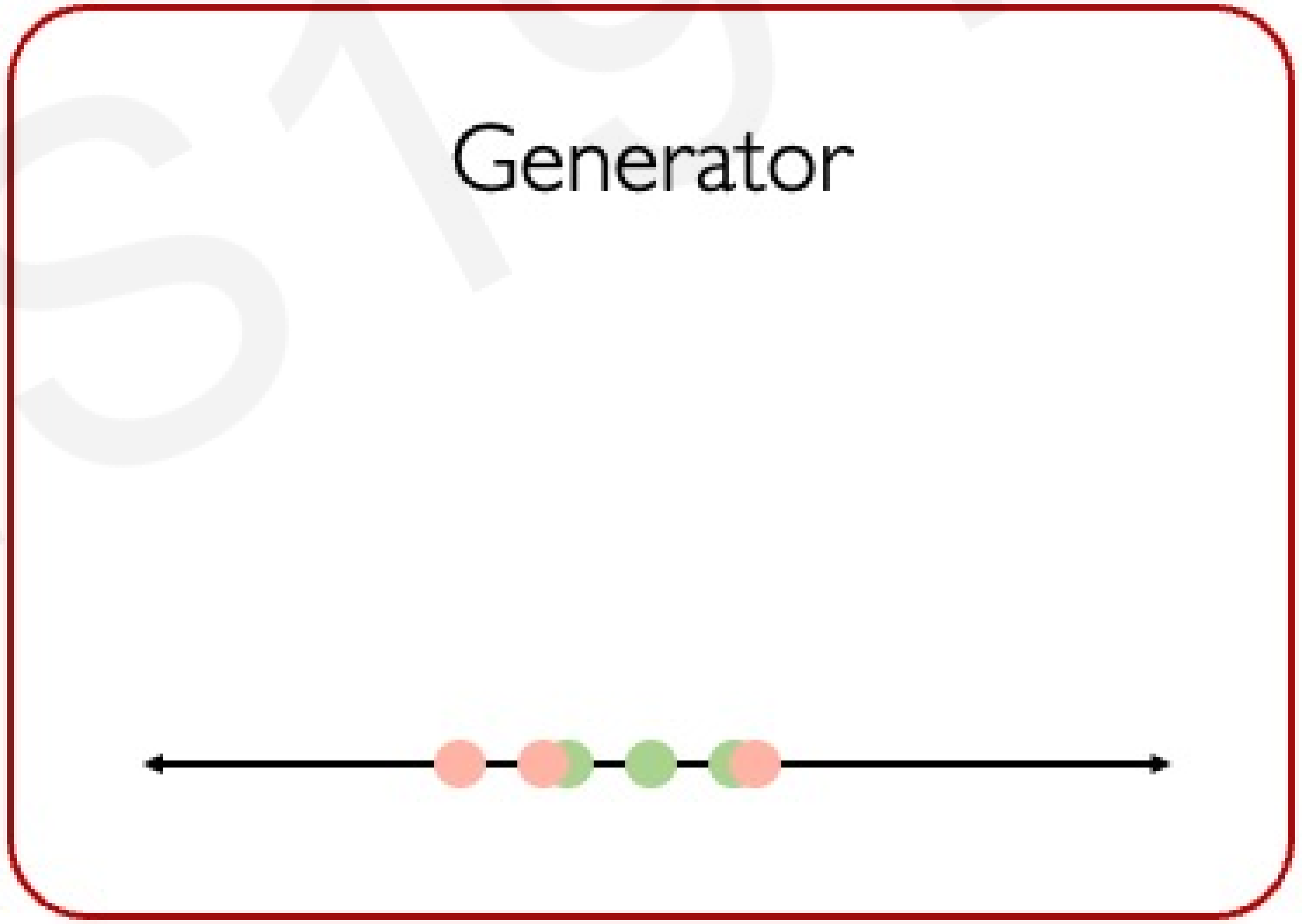

Real data Fake data

Intuition behind GANs

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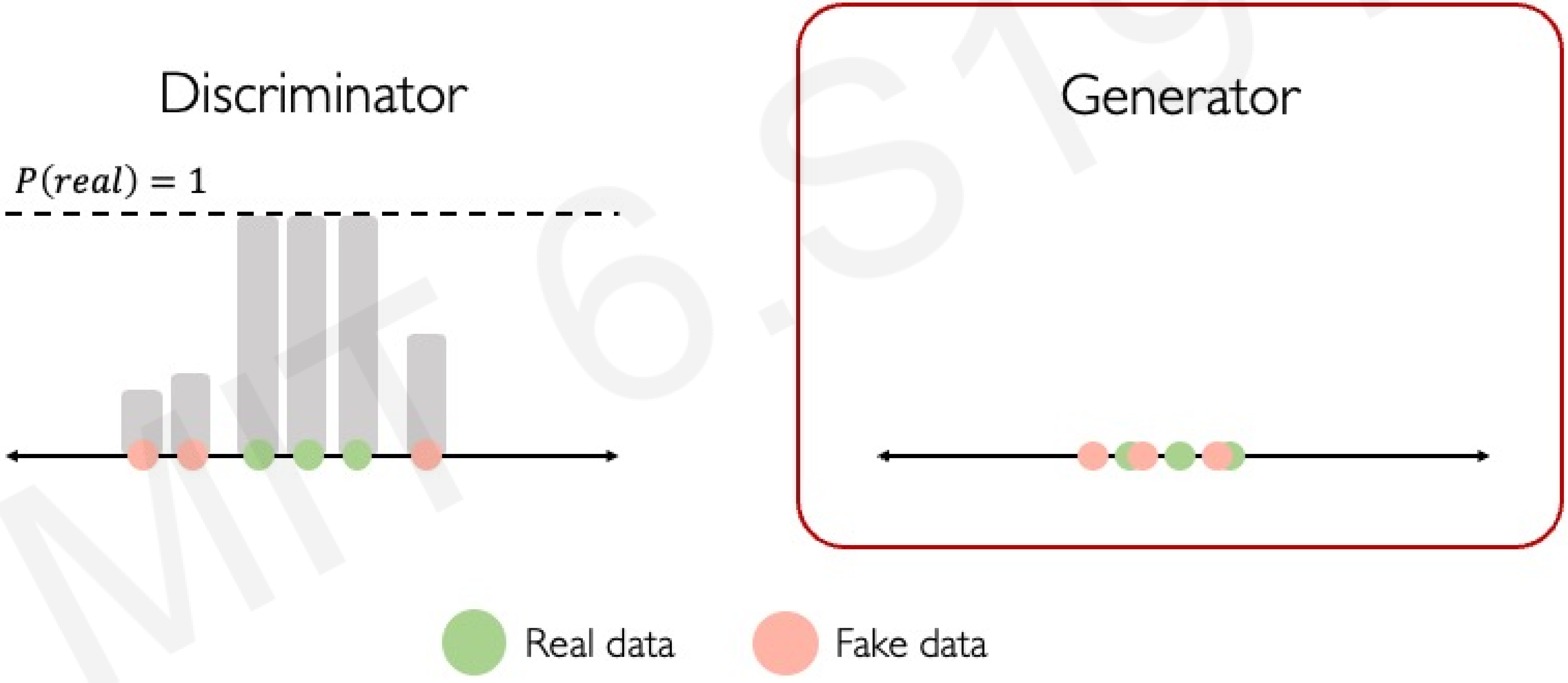
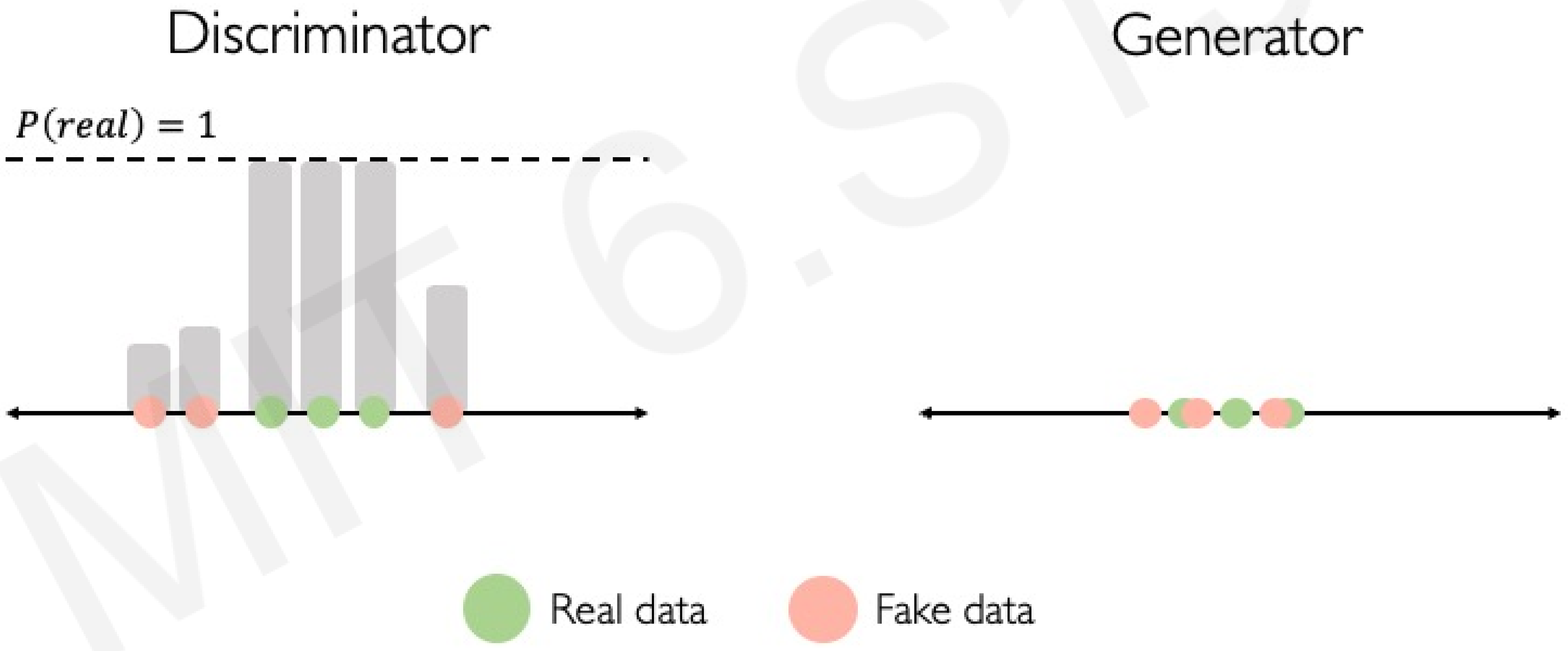
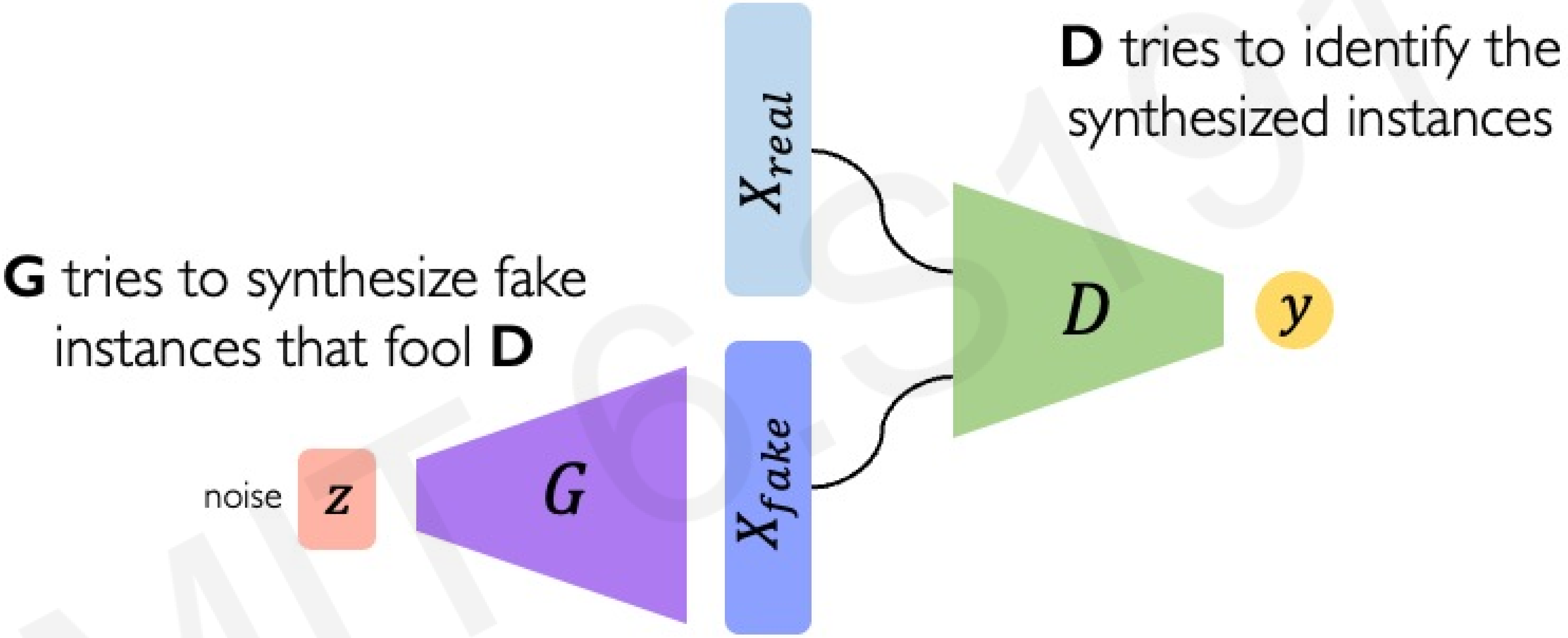


Intuition behind GANs

Discriminator tries to identify real data from fakes created by the generator.
Generator tries to create imitations of data to trick the discriminator.



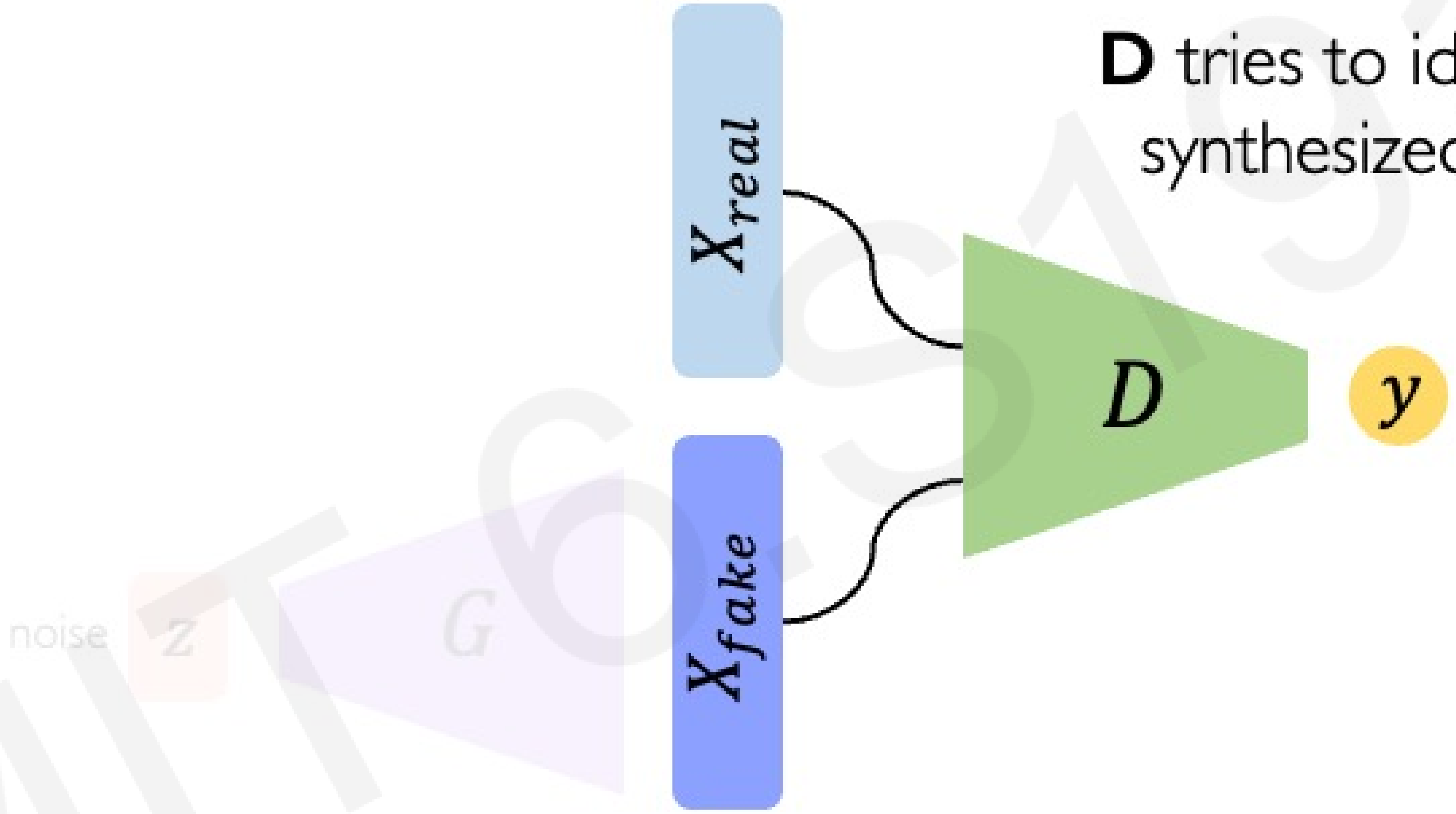
Training GANs



Training: adversarial objectives for **D** and **G**

Global optimum: G reproduces the true data distribution

Training GANs: loss function

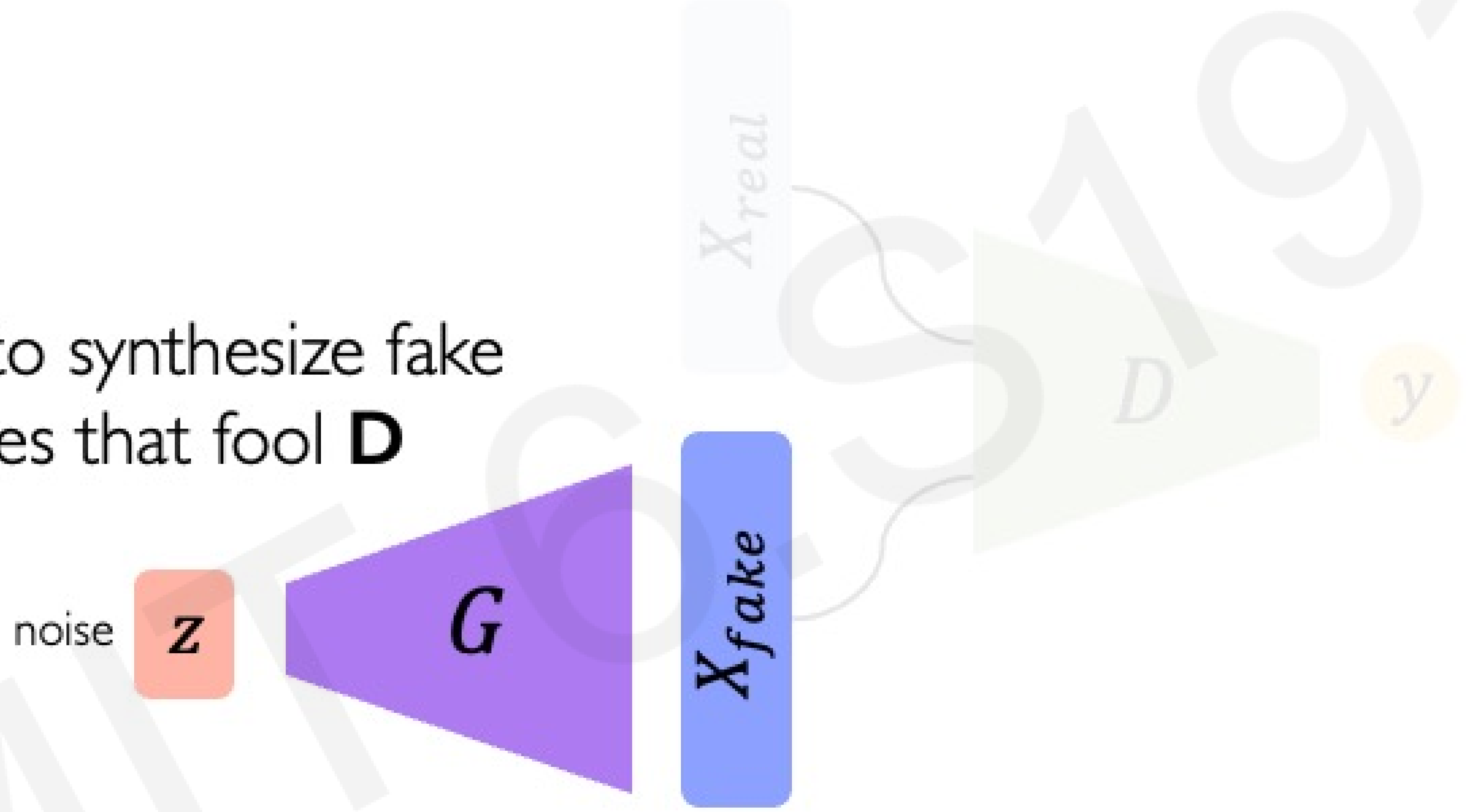


D tries to identify the synthesized images

$$\arg\max_{D} \mathbb{E}_{\mathbf{z},\mathbf{x}}\left[\, \underbrace{\log D(G(\mathbf{z}))}_{\textbf{Fake}} + \underbrace{\log\left(1 - D(\mathbf{x})\right)}_{\textbf{Real}} \,\right]$$

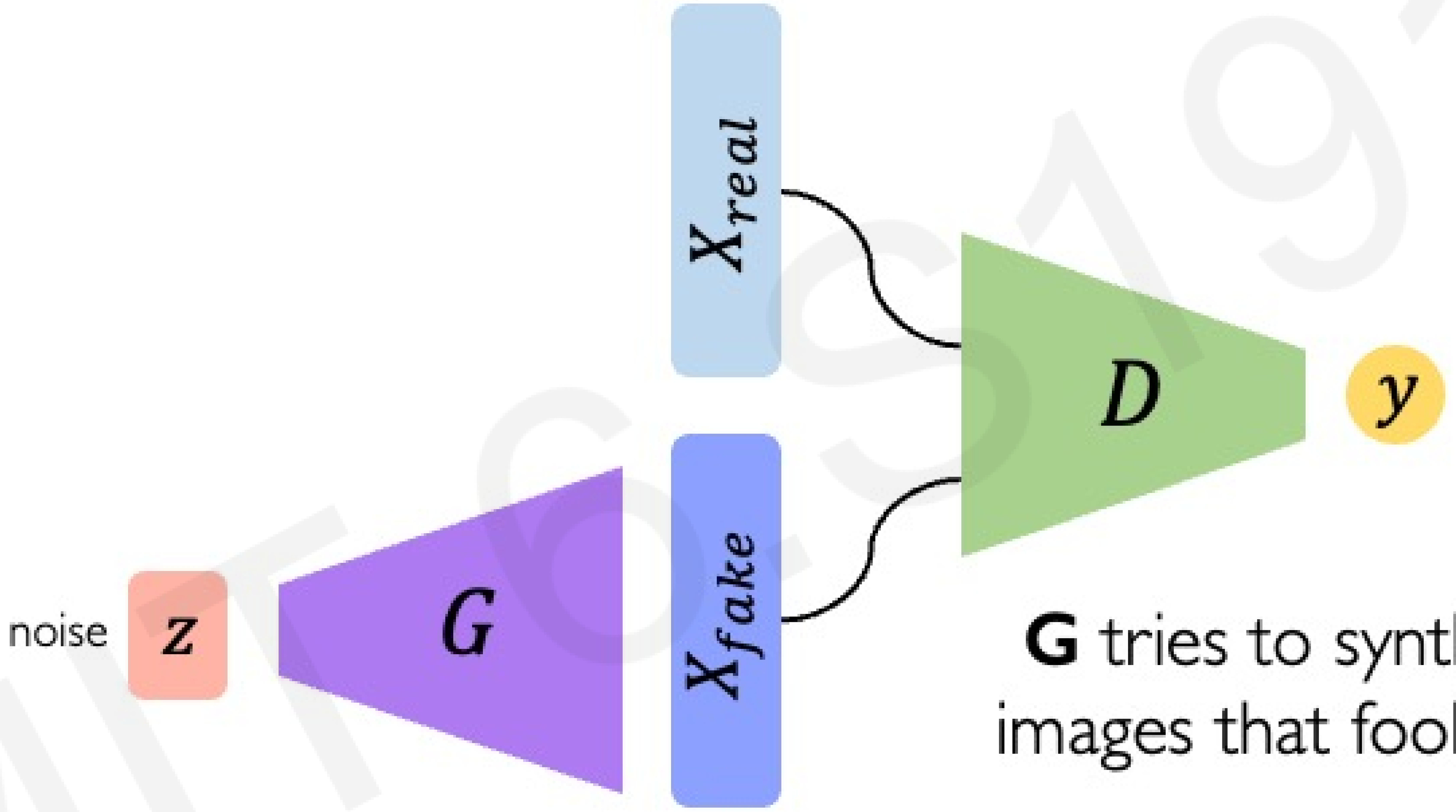
Training GANs: loss function

G tries to synthesize fake images that fool **D**



$$\arg\min_{G} \mathbb{E}_{\mathbf{z},\mathbf{x}}\left[\, \log D(G(\mathbf{z})) + \log\left(1 - D(\mathbf{x})\right) \,\right]$$

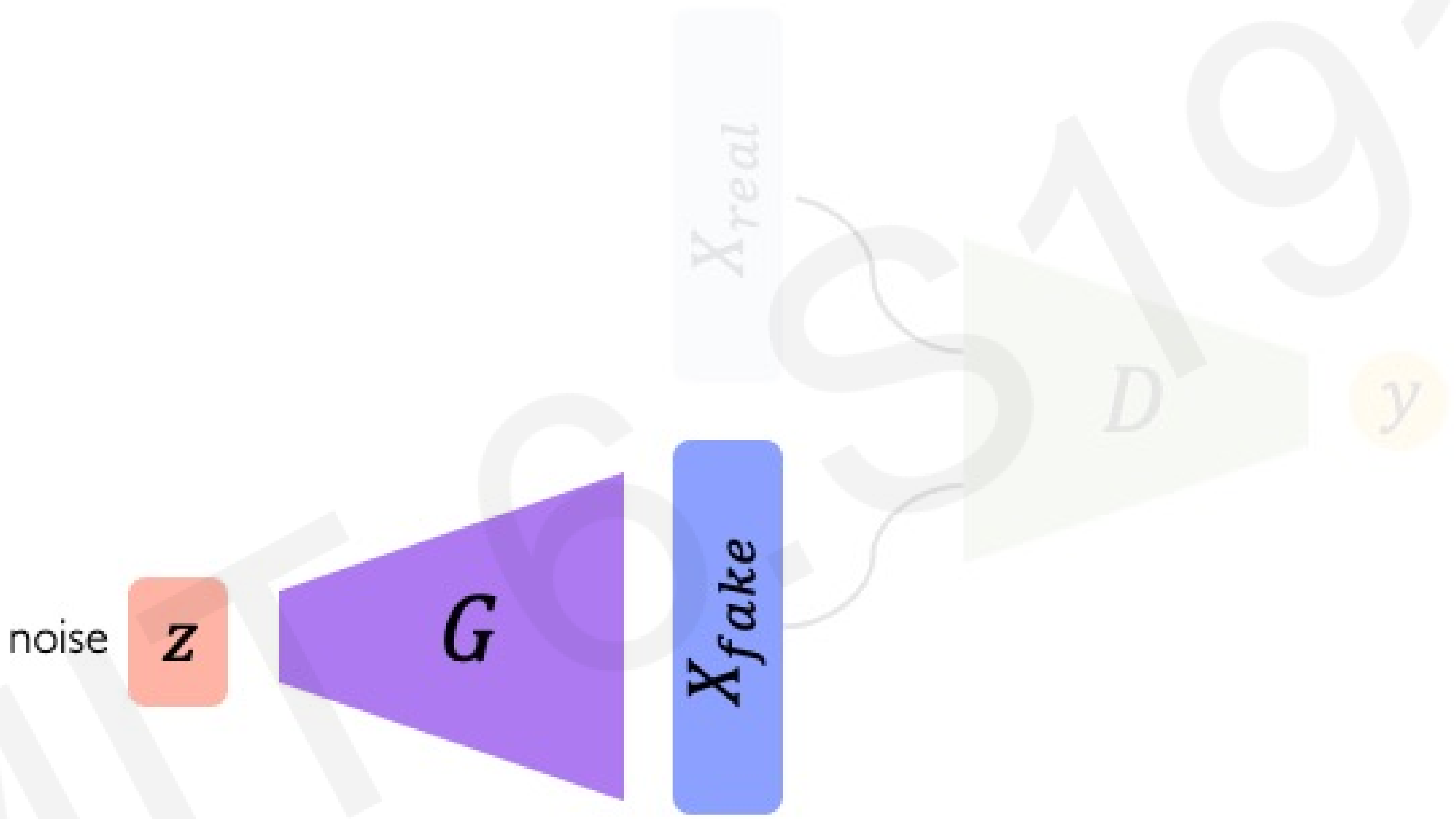
Training GANs: loss function



G tries to synthesize fake images that fool the *best* **D**

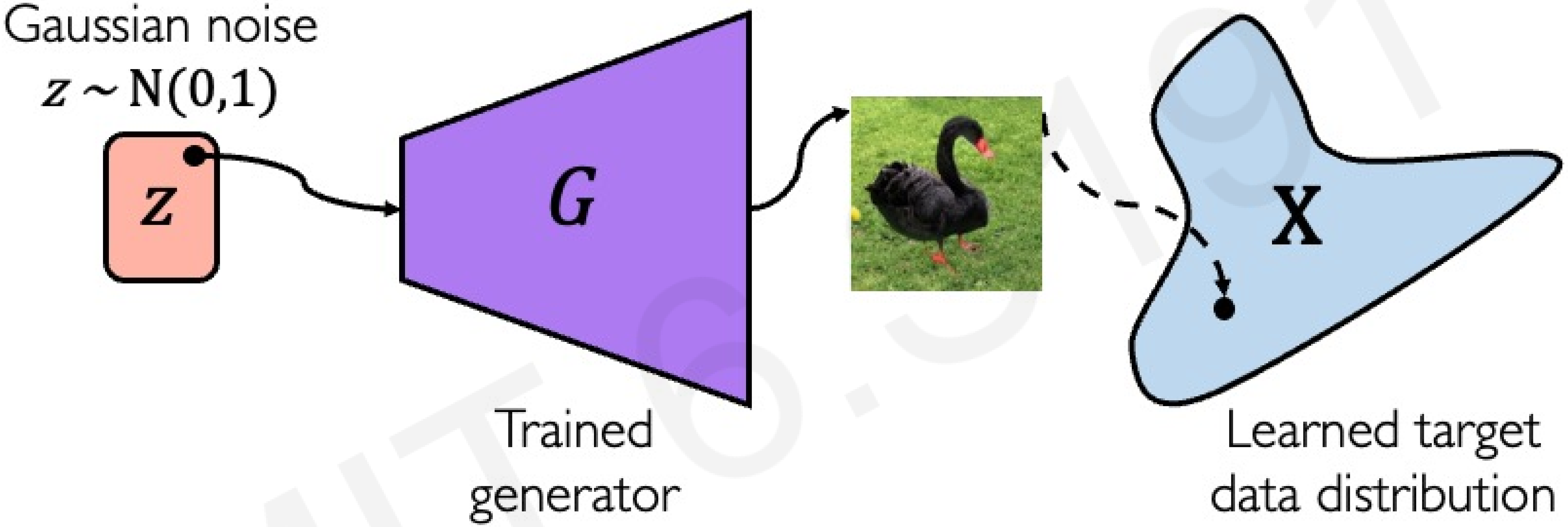
$$\arg\min_{G}\max_{D}\ \mathbb{E}_{\mathbf{z},\mathbf{x}}\left[\ \log D(G(\mathbf{z})) + \log\left(1 - D(\mathbf{x})\right)\ \right]$$

Generating new data with GANs

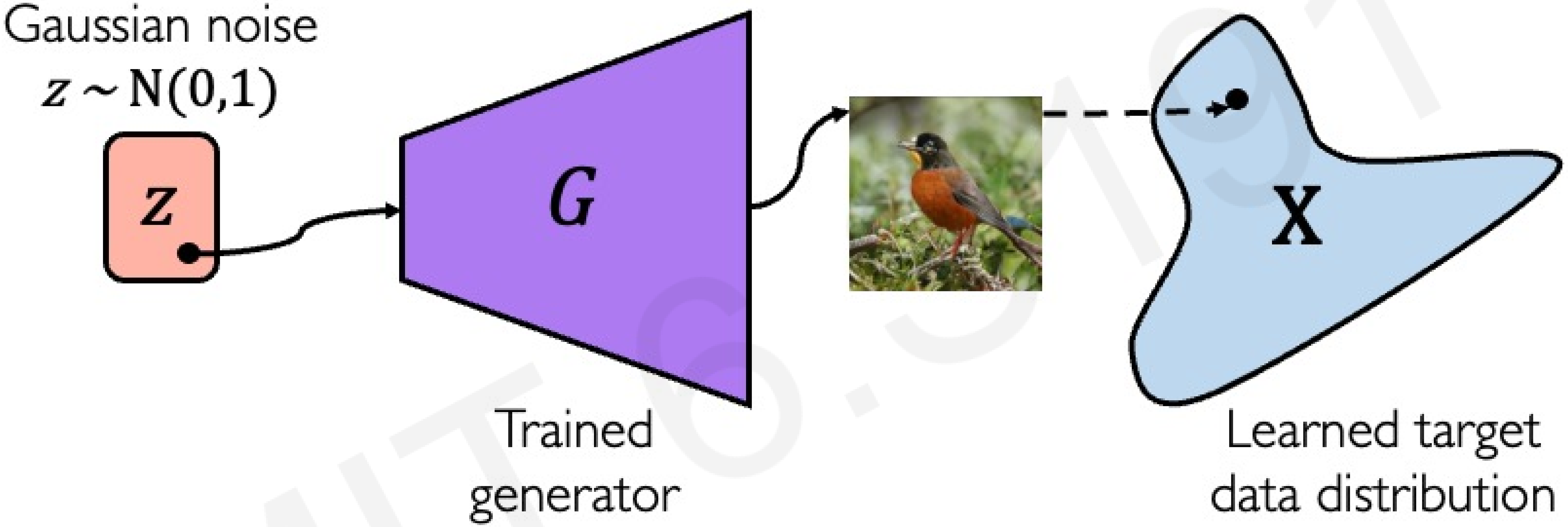


After training, use generator network to create **new data** that's never been seen before.

GANs are distribution transformers



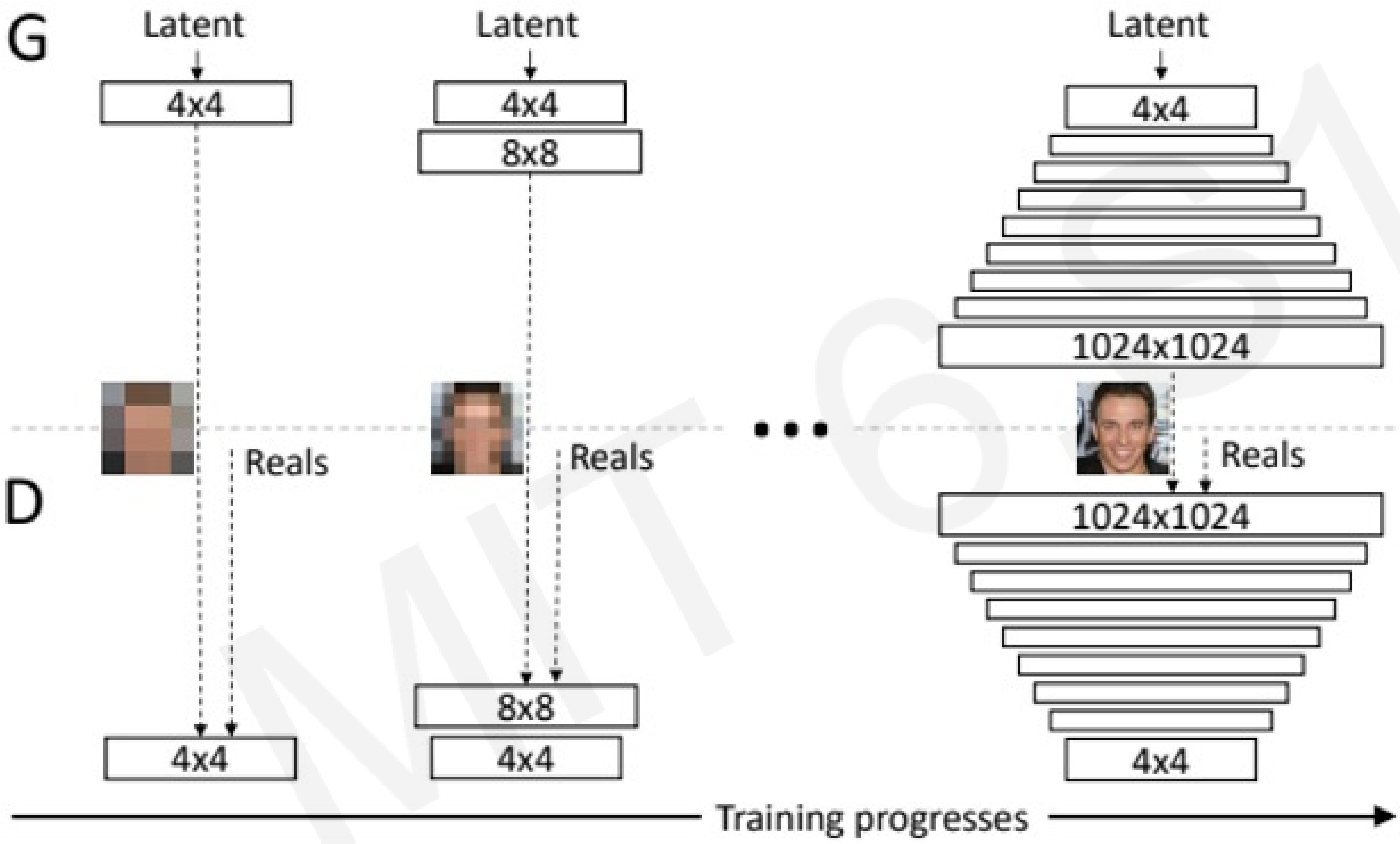
GANs are distribution transformers



GANs are distribution transformers

GANs: Advances and Applications

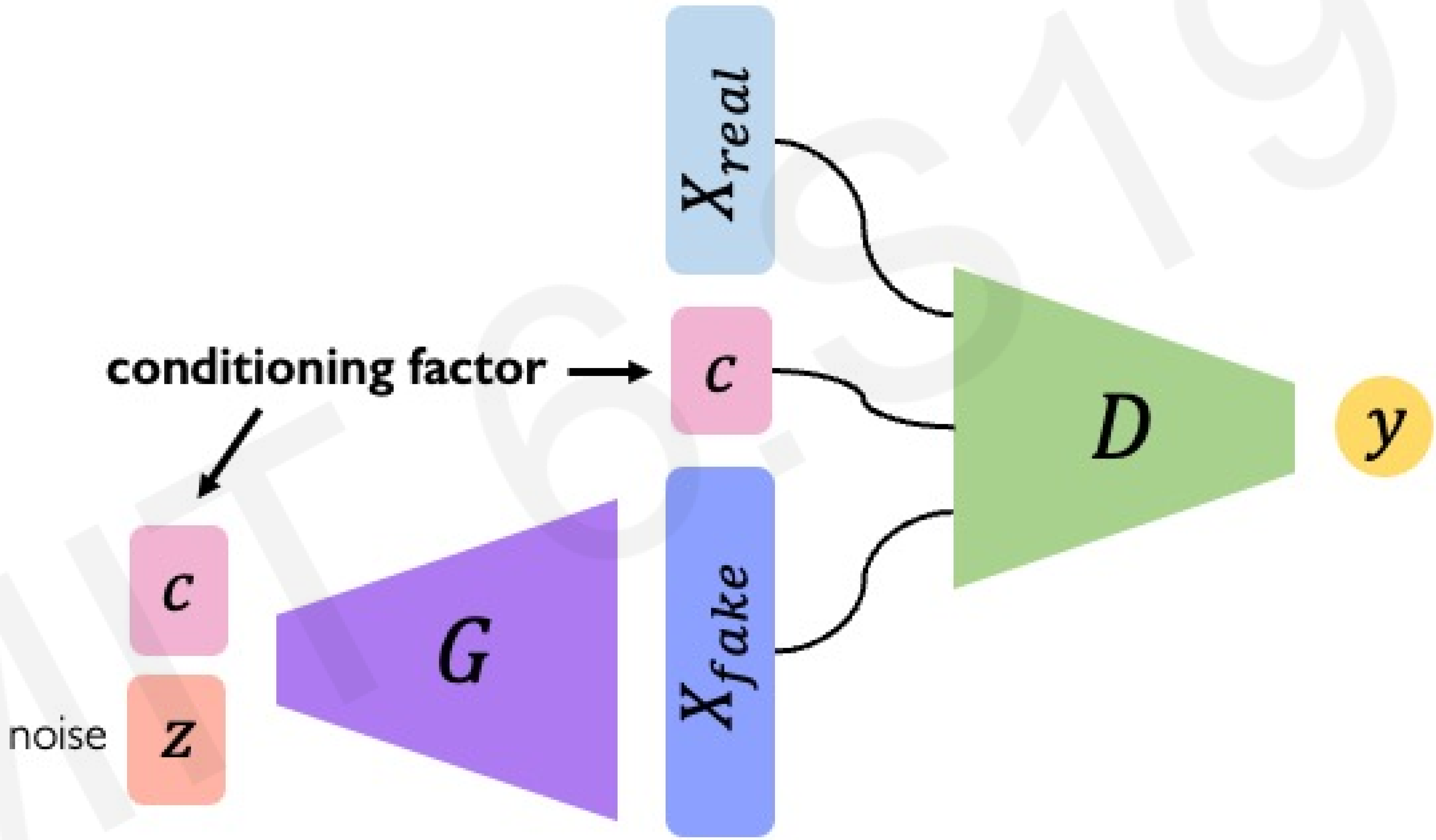
Progressive growing of GANs



Progressive growing of GANs: results

Conditional GANs

What if we want to control the nature of the output, by **conditioning** on a label?

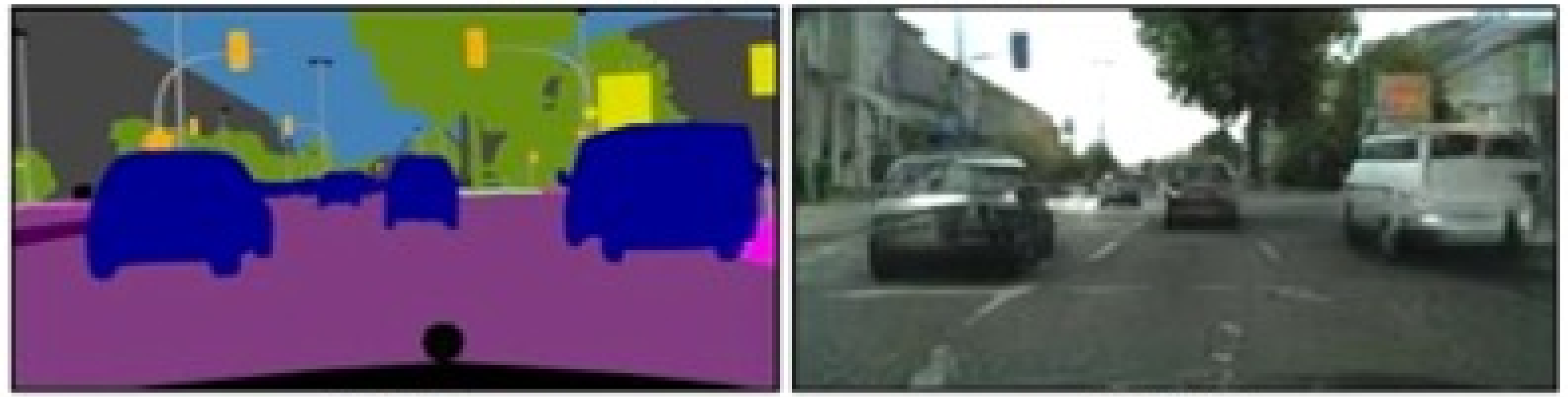


Conditional GANs and pix2pix: paired translation

The discriminator, D, classifies between fake and real **pairs**.
The generator, G, learns to fool the discriminator.

Applications of paired translation

Labels to Street Scene



input output

Paired translation: results

Map → Aerial View | **Aerial View → Map**

input **output** **input** **output**

CycleGAN: domain transformation

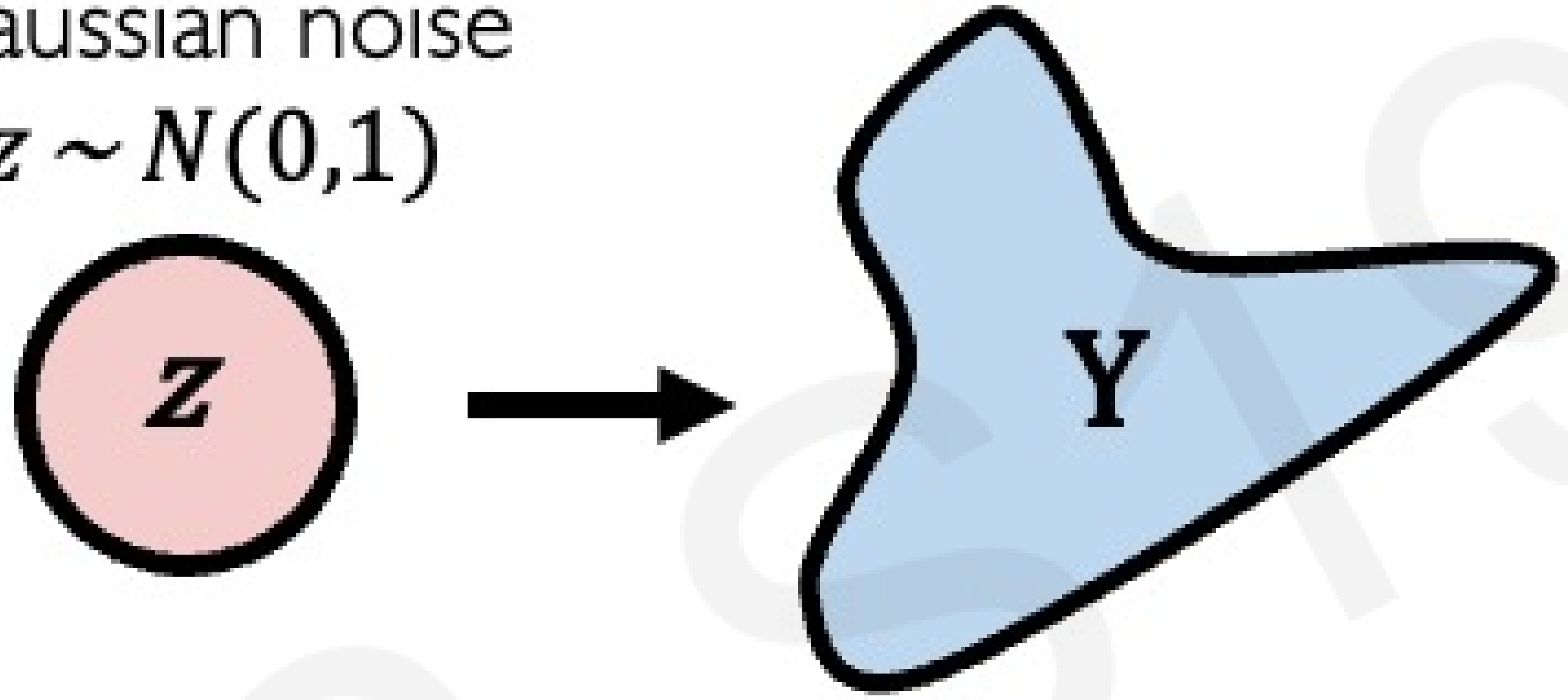
CycleGAN learns transformations across domains with unpaired data.

$$
\begin{tikzcd}
D_X & & D_Y \\
X \arrow[u, dotted] \arrow[rr, bend left, "G"] & & Y \arrow[u, dotted] \arrow[ll, bend left, "F"]
\end{tikzcd}
$$

Distribution transformations

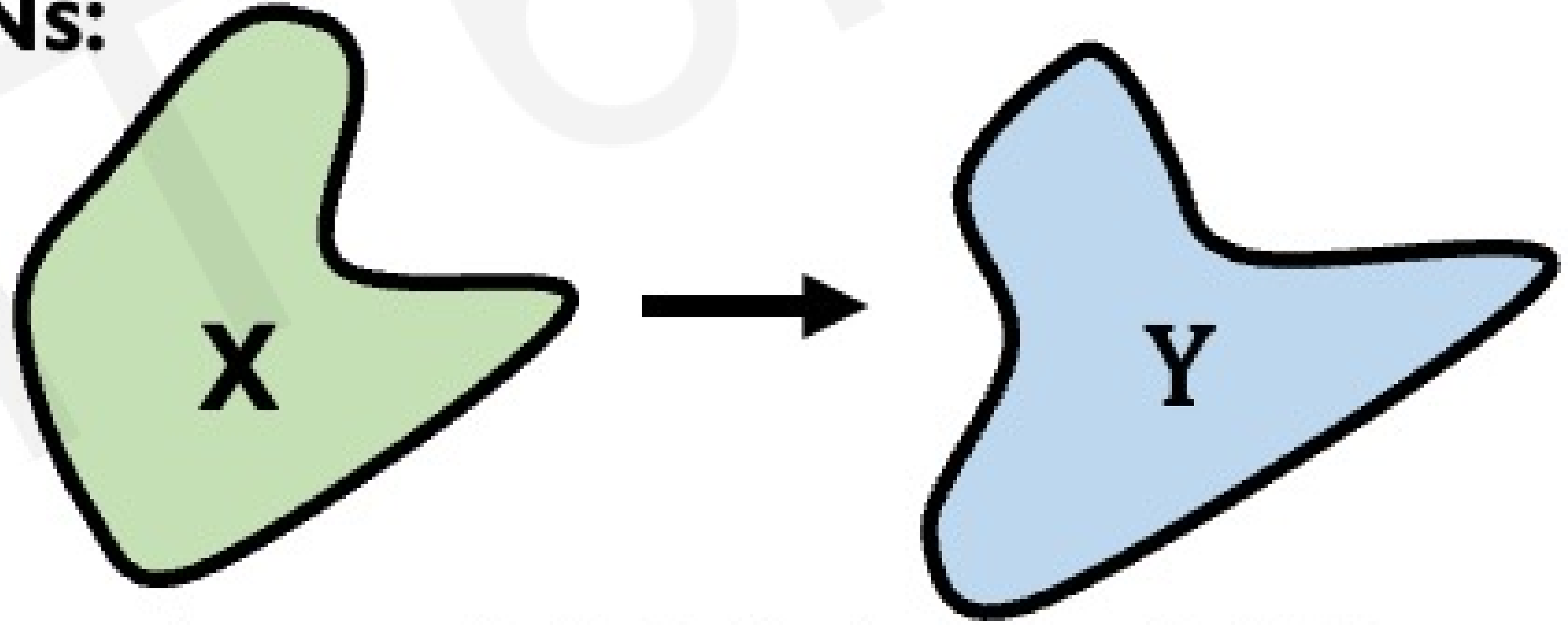
GANs:

Gaussian noise
$z \sim N(0,1)$



Gaussian noise → target data manifold

CycleGANs:



data manifold X → data manifold Y

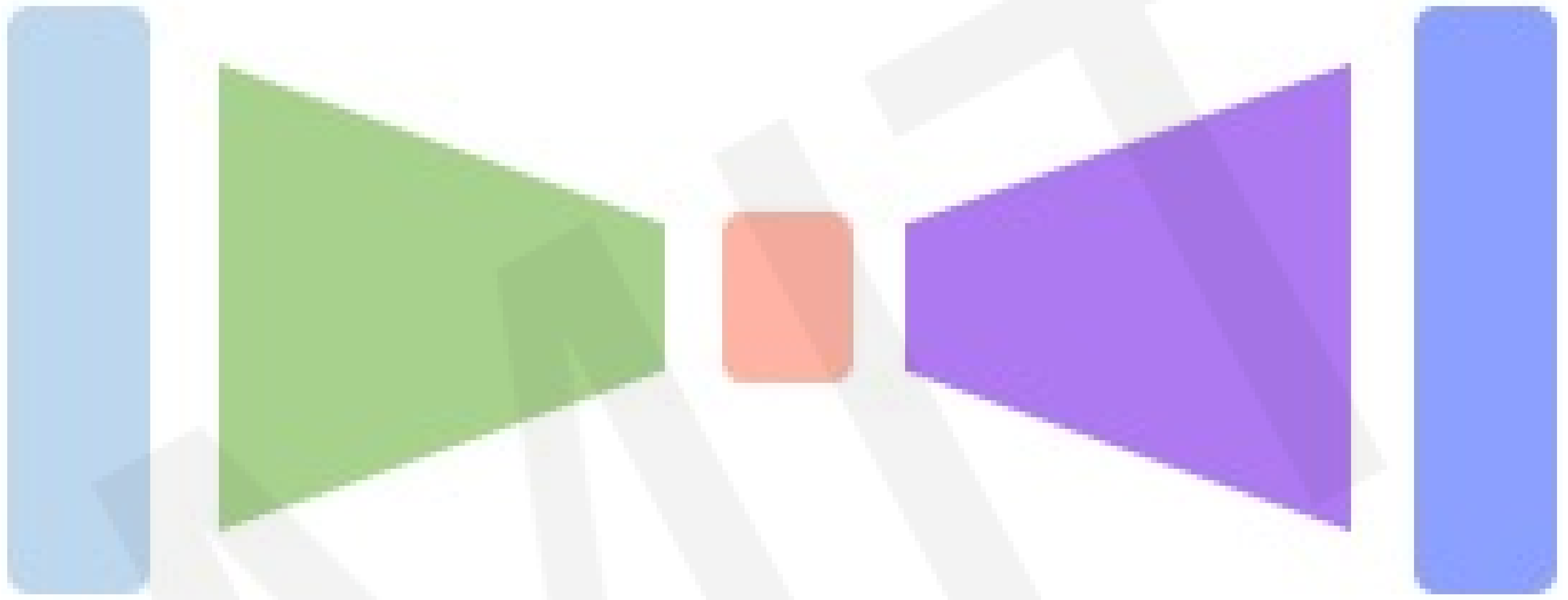
CycleGAN: transforming speech


Original (Amini)
Synthesized (Obama)
CANNY AI
0:02.560
0:02.571

Deep Generative Modeling: Summary

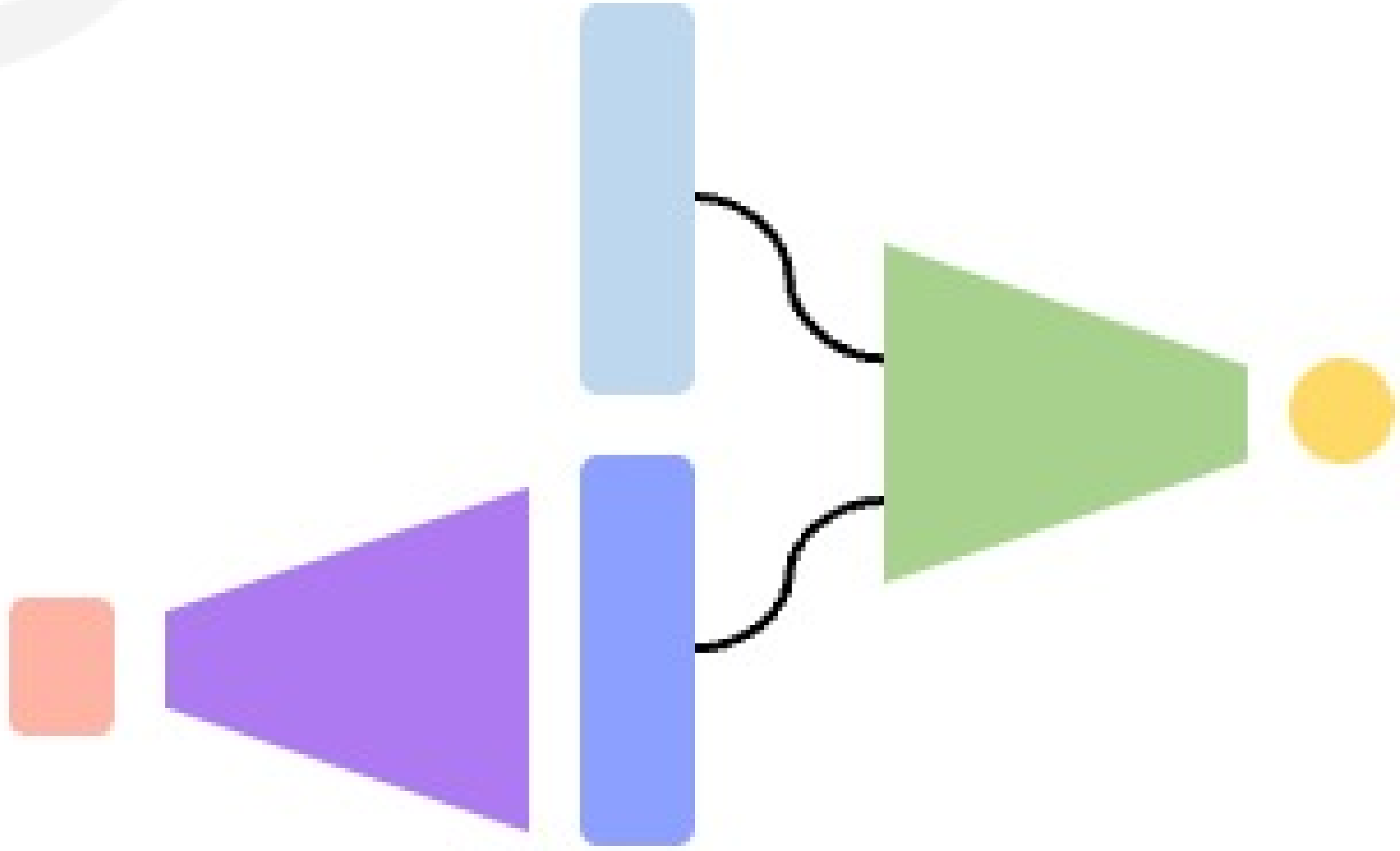
Autoencoders and Variational Autoencoders (VAEs)

Learn lower-dimensional **latent space** and **sample** to generate input reconstructions



Generative Adversarial Networks (GANs)

Competing **generator** and **discriminator** networks



Diffusion Models

...more to come in Lectures 6 and 10!

MIT
Introduction to Deep Learning

Lab 2: Facial Detection Systems

Link to download labs:
introtodeeplearning.com#schedule
github.com/MITDeepLearning/introtodeeplearning

1. Open the lab in Google Colab
2. Start executing code blocks and filling in the #TODOs
3. Need help? Come to 32-123!